

Structural Analysis III
Qualitative Analysis

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1. Introduction

1.1 *Background*

The ability to ‘see’ and interpret structural behaviour is a core ability of a structural engineer. At the initial stage of a structural scheme design, we are not interested in numbers, or amounts, only the sense of a load effect. Some examples of what we mean by sense are:

- Is there tension on the top or bottom of a beam?
- Does the tip of a cantilever deflect up or down?
- Is the moment reaction clockwise or anti-clockwise?

Getting this level of analysis right is not only the first step, but the most important step. If we don’t get this level right, then the answers to a more complicated analysis will be meaningless.

The ability to get the right answers to this level is called *Structural Intuition*. The better your structural intuition, the better you will be a designer. This ability reduces errors both in design practice but also whilst in college: since you will already ‘see’ the answer it is easier to catch errors in calculations.

1.2 Reading Material

Some good books on structural behaviour are:

- Brohn, D., *Understanding Structural Analysis*, 4th Edn., New Paradigm Solutions, 2005.
- Jennings, A., *Structures: from theory to practice*, Spon Press, 2004.
- Ji, T., and Bell, A., *Seeing and Touching Structural Concepts*, Taylor & Francis, 2008.
- Hilson, B., *Basic Structural Behaviour: Understanding Structures from Models*, Thomas Telford, 1993.
- Pippard, A.J.S., *The Experimental Study of Structures*, Edward Arnold & Co., London, 1947.
- I.Struct.E., *Qualitative Analysis of Structures*, London, 1989.

Due to its importance, the Ove Arup Foundation sponsored the report: *The Teaching of Structural Analysis* by Prof. Ian May and Dr. David Johnson. It is accessible here: <http://www.jbm.org.uk/uploads/StructuralAnalysiswithCover.pdf>.

A summarized version of the report appeared in *The Structural Engineer*, Vol.81, No.7, 2003, p.33-37, available at this link:

<http://www.istructe.org/thestructuralengineer/Abstract.asp?PID=7904>

1.3 Software

In developing your structural intuition, it is very helpful to model structures using an appropriate computer program – especially when the structure behaves counter-intuitively. Most structural analysis programs today are extremely complex with many options and capabilities and this can often obscure the modelling process.

An appropriate program (for a few reasons) is *LinPro* – freely available from www.line.co.ba. You should install *LinPro* on your own computer. Also, it is installed on the computers in Rm 392.

The program is intuitive to use and comes with a reasonable help file. If you have any difficulties using the program, please ask the lecturer.

Another program for the analysis of trusses is *TrussMaster*, developed by the lecturer for the purposes of teaching structural behaviour of trusses. This is available on the college computers, and a User Manual is also available at www.colincaprani.com.

2. Qualitative Analysis Techniques

2.1 *Introduction*






Qualitative Analysis is not a linear process. For some problems we might start with reactions and proceed through bending moments to a displaced shape, whilst for others we may begin with a displaced shape, work out reactions, and then find the bending moment diagram. The approach to use will depend on the problem and there are set rules or procedures that you can follow to be guaranteed to arrive at the correct solution.

On a more positive note, since the structure will only behave in one distinct manner, there can only be one correct solution. By definition therefore, incorrect solutions will contain inherent incompatibilities. For example, all aspects of a frame's solution may agree (e.g. reactions, bending moment diagram, etc.), but it may require a rigid joint to have different rotations. Since this is impossible, we know that this cannot be the right answer. Therefore if we have a proposed solution, we must ensure that it does not violate any of the conditions of structural behaviour. If it does, then it is not the correct solution. In other words, your answer will tell you if it is wrong or not!

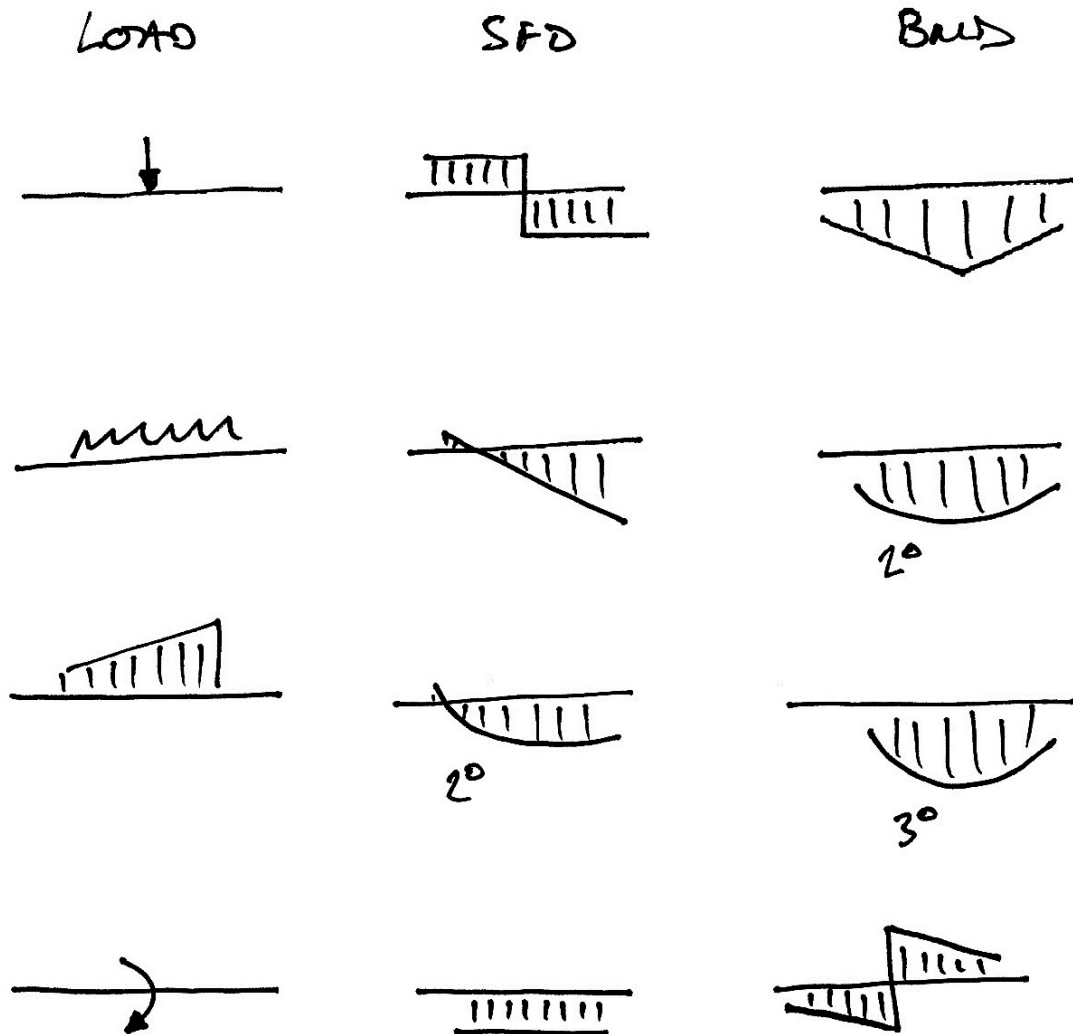
2.2 Conditions of Structural Behaviour

There are certainties about structural behaviour that we can rely on when attempting to analyse a structure. Most of these are plainly obvious, but a few may not be.

1. Remember the very basics: moment = force \times distance.
2. Know your support types and the type of restraint they offer:

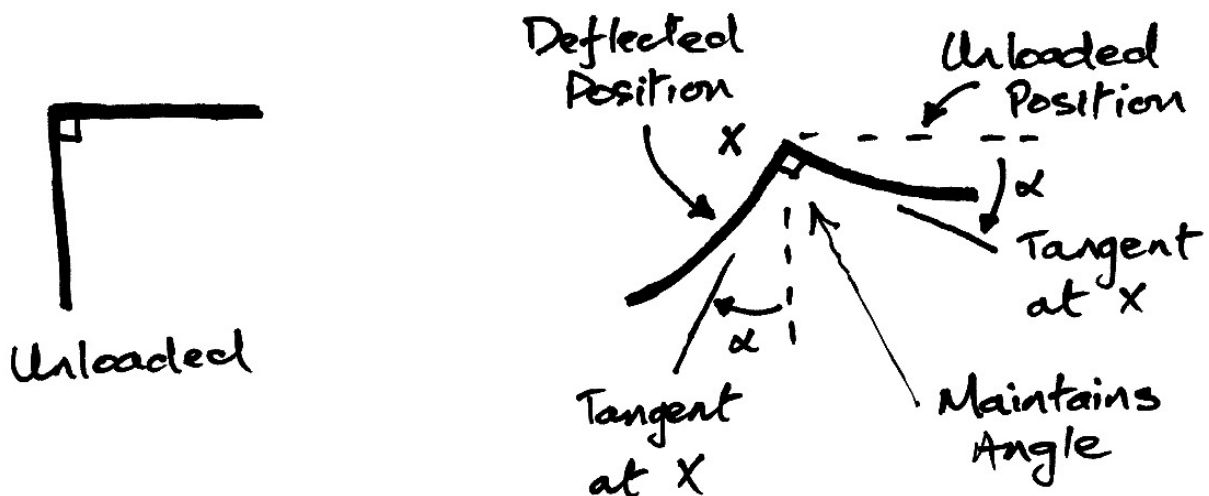
Symbol	Name	Movements
	Roller (Horizontal)	δ_x δ_y θ
	Vertical Roller	δ_x δ_y θ
	Pin	δ_x δ_y θ
	Fixed	δ_x δ_y θ
	Vertical Support (beam continuous over the support and can rotate)	δ_x δ_y θ

3. Recall the shapes of BMD and SFD under the different types of loading (rectangular, triangular, parabolic).

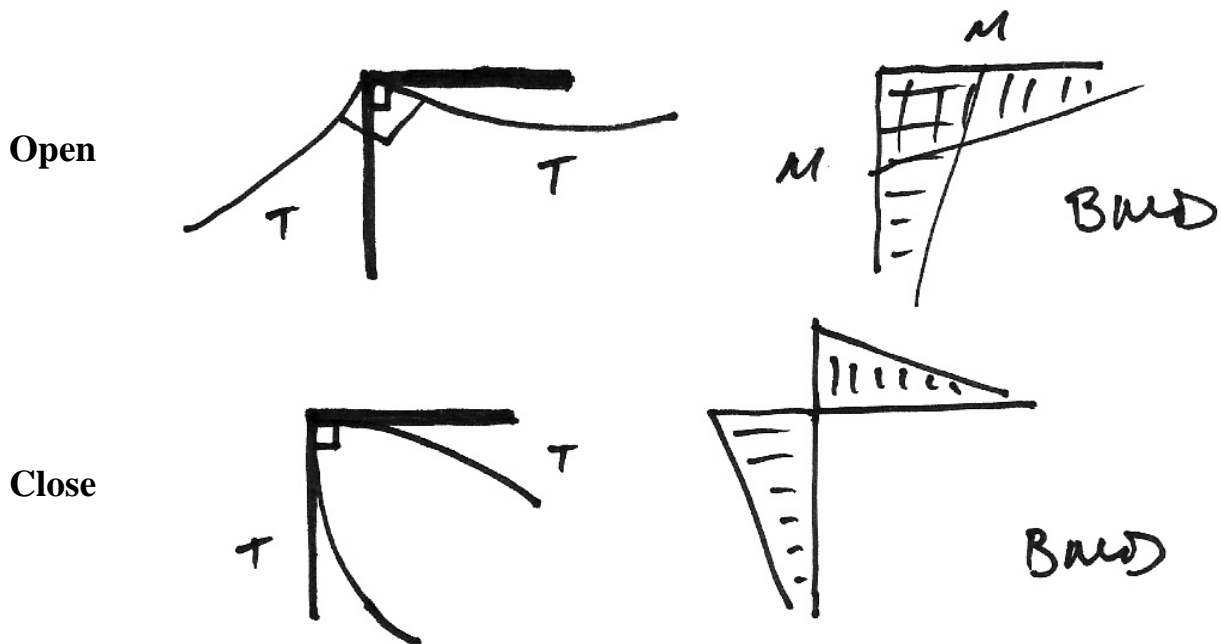


4. Remember: shear is rate of change of moment.
5. No transverse load or end shear force on a frame member means there is constant BM along the member (constant may equal zero).
6. There is zero bending moment at a hinge.

7. Members with no bending moments remain straight (i.e. no bending), but may still move.
8. Always draw the bending moment diagram on the tension face of the member. to be consistent with our convention
9. Remember, fixed supports will have a moment reaction, pinned supports will not, though there may be an external moment applied at a pinned support.
10. For unbraced frames, only symmetrical such frames symmetrically loaded will not sway.
11. Keep in mind: deflections are always small and we neglect the self weight of the structures – only analyse for the loads shown.
12. Deflected shapes are always very smooth curves, except at a hinge.
13. Rigid joints in frames must keep the same angle as they rotate.

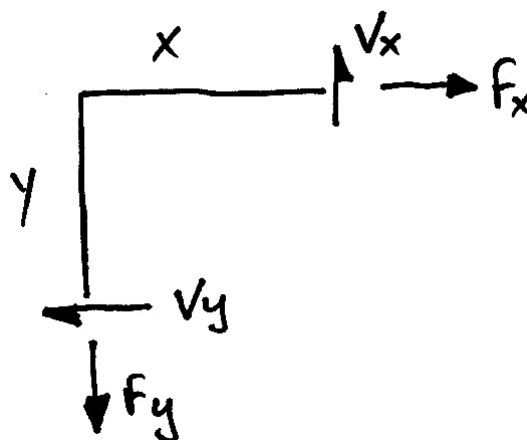


14. Rigid joints can only open or close:

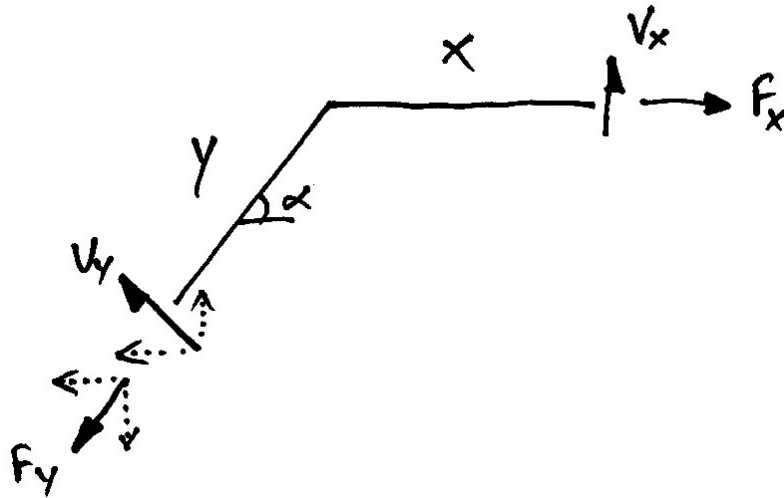


15. At a rigid joint with two members, there is only one value of moment (M above). (There is one rare exception to this rule)

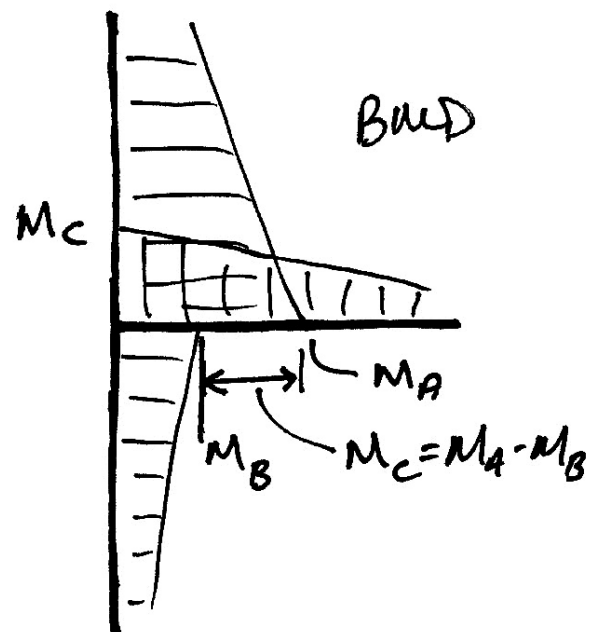
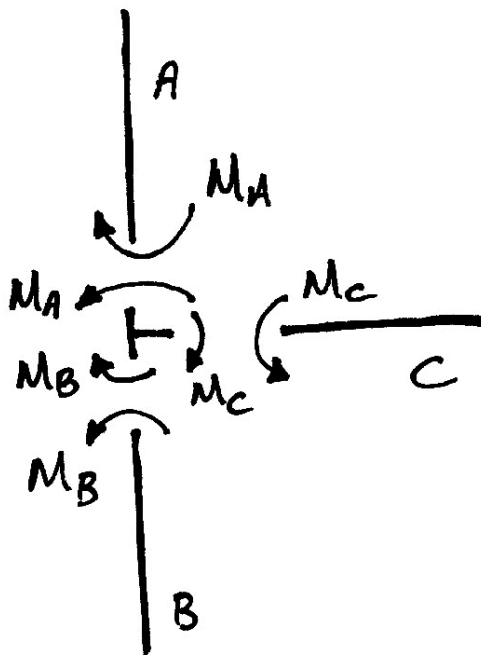
16. At a right-angle rigid joint, the shear becomes the axial and the axial becomes the shear in the alternate members. Just use $\sum F_x = 0$ and $\sum F_y = 0$ to see why.



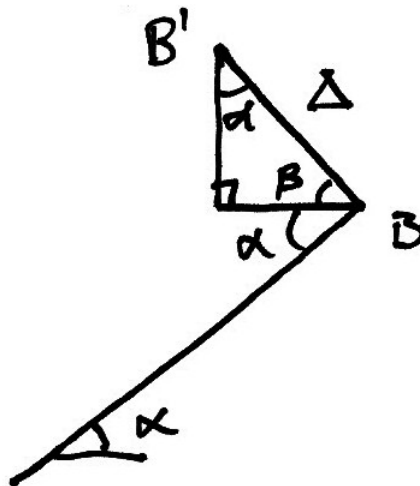
17. This is not the case for oblique-angle joints:



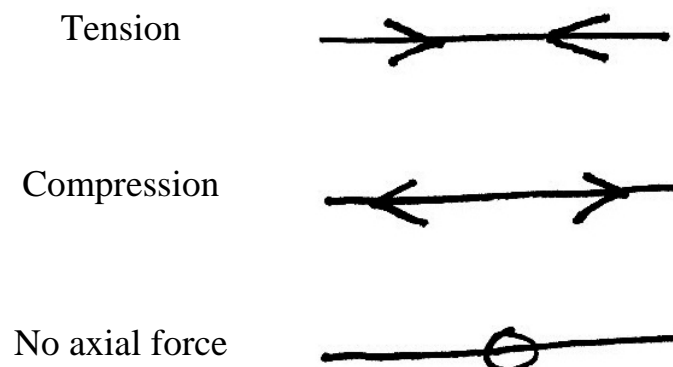
18. When more than two members meet at a rigid joint, the joint must be in equilibrium. This means that for the joint below, $M_A = M_B + M_C$. A further implication of this is seen in the BMD: there is a step in the bending moment for member AB at the joint of value $M_C = M_A - M_B$.



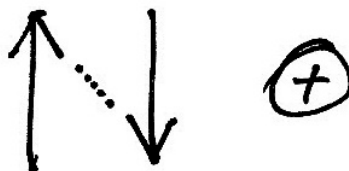
19. For frames, we normally neglect axial deformation. This means that members cannot change length and because deflections are small, this means that the member's joints must move perpendicular to the line of the member. For example, below B can only move along the line BB' .



20. Trusses do not have bending moment diagrams.
21. Remember the axial force sign convention:



22. Positive shear force sign convention makes the letter 'N': up on the left, down on the right:



2.3 *Methods to Aid Solution*

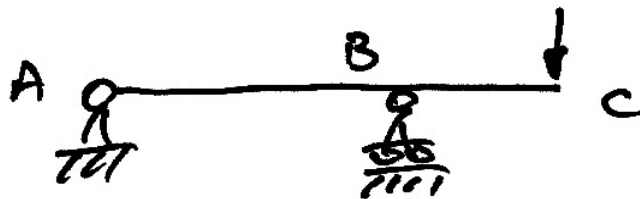
The following are some methods that may help you carry out the analyses:

1. To find a support reaction, *Remove the Restraint* offered by the reaction and draw the deflected shape of the resulting structure. Apply the support reaction in such to as to bring the structure back to where it should be.
2. Use *Points of Certainty* – where you know the deflected position, for example at a support the deflection is zero, and usually the structure moves away from the applied load (though there are rare exceptions).
3. For more complex structures, remove excess members/supports/joints and reintroduce one at a time and observe the effect each additional feature has.

2.4 Example 1

Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.

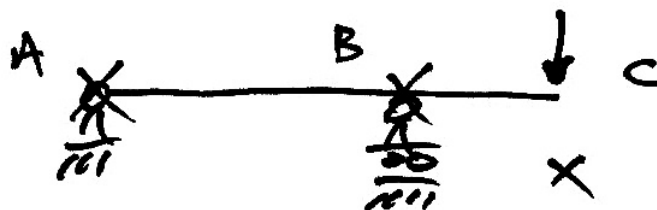


Solution

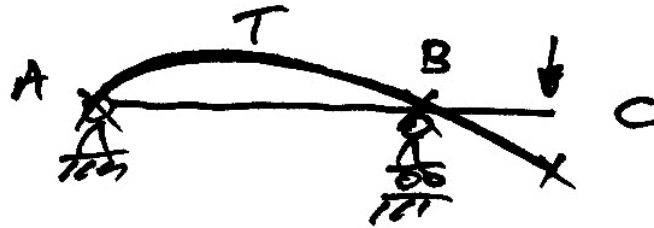
Firstly we identify the *Points of Certainty*:

- It cannot move horizontally or vertically at A;
- It cannot move vertically at B;
- It will probably move downwards at C away from the load.

This gives the following points through which the deflected shape must pass:



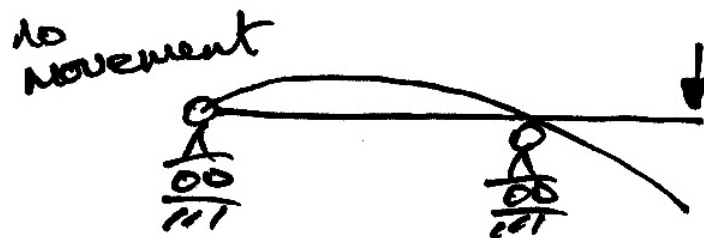
Noting that the deflected shape is always a smooth curve (except at hinges, of which there are none here), we join the three points with a smooth curve:



Also, we know there is tension (“T” above) on the outside of the curve and so we include this in our drawing. This helps inform us of the bending moment diagram – always draw it on the tension face.

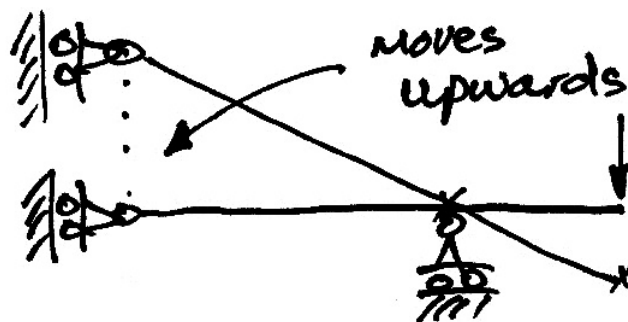
To find the direction of the reactions, we will remove each restraint in turn and follow the above steps to see how the beam deflects when the restraint is removed.

- For H_A :



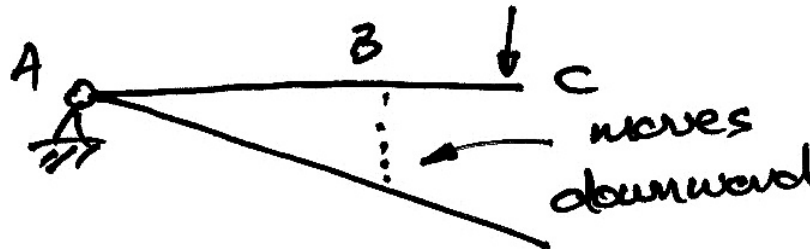
Since there is no movement of the beam when H_A is released, $H_A = 0$.

- For V_A :



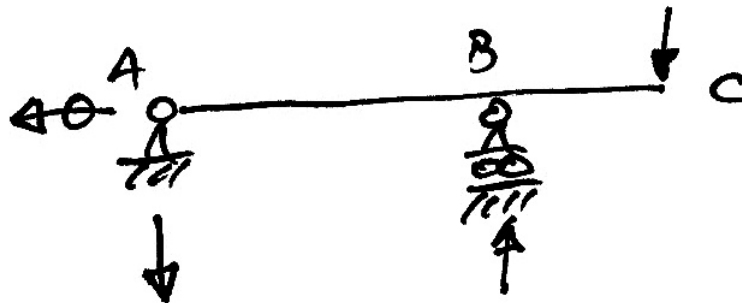
Since when V_A is removed it moves upwards, it must be that V_A acts downwards in the actual structure to keep the beam at A where it must remain.

- For V_B :

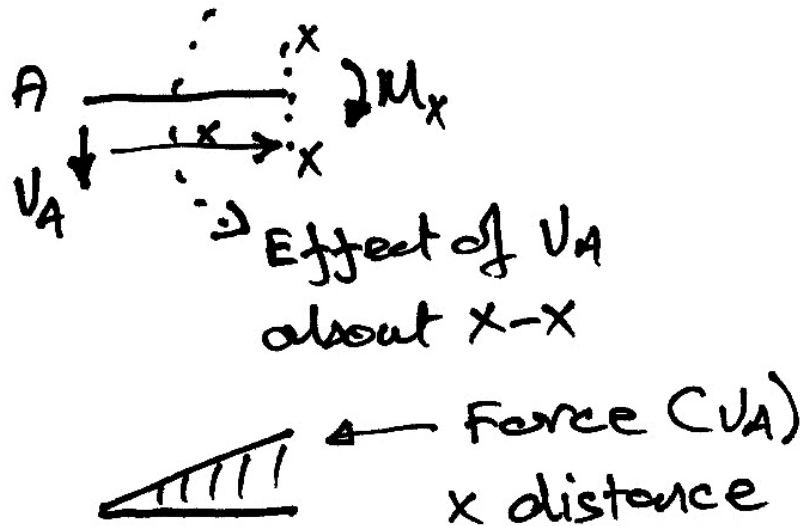


Since when V_B is removed the beam moves downwards, V_B acts upwards in the actual structure to ensure that B remains where it should.

Thus the reactions are:

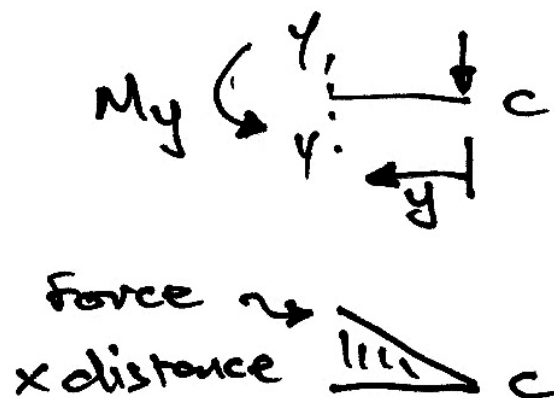


With this information it now becomes easier to establish the bending moment and shear force diagrams. Starting with the bending moment diagram for the portion AB of the beam, we take a cut somewhere to the right of A :

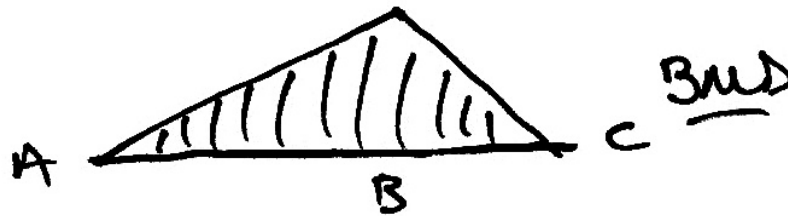


As may be seen, the effect of V_A is to cause an anti-clockwise rotation of the segment which must therefore be resisted by a clockwise internal bending moment M_X , as shown. This means (since the arrow comes from the tension face) that tension is on the top of the beam, and is increasing as the distance increases (the force remaining constant).

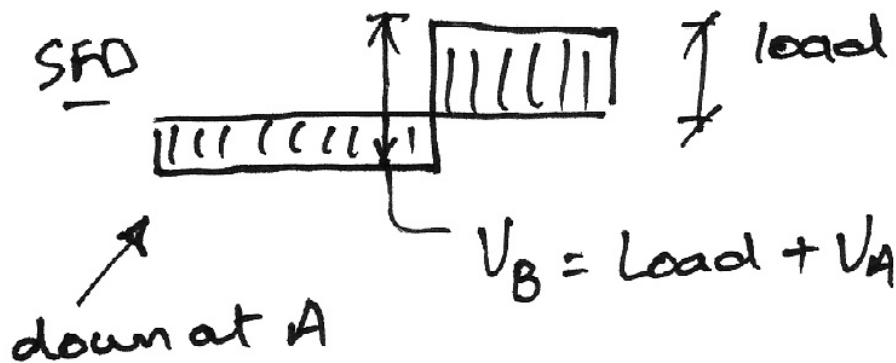
Similarly, we examine the portion BC:



There are no applied moments, so the bending moment diagram does not have any steps in it. This means that the two portions that we have identified above must meet over B to give:



The shear force diagram is easy to construct by just following the forces: moving left to right it is down at A then up at B over the line to a height equal to the applied force. The total height at B is the vertical reaction at B and this must sum to the total downward forces at A and C :

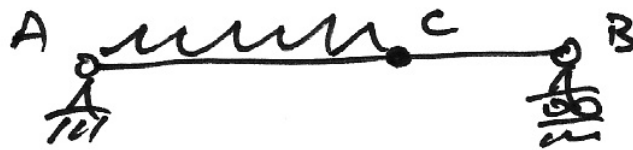


Note also that since $V = dM/dx$, we see that the negative shear corresponds to a negative slope in the BMD

2.5 Example 2

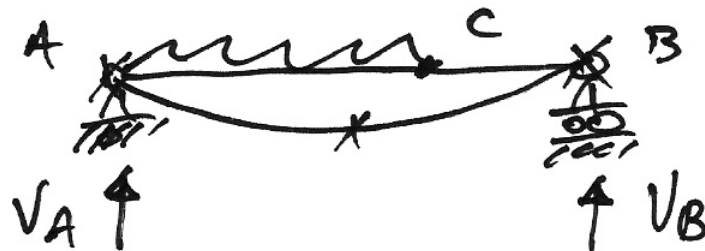
Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.



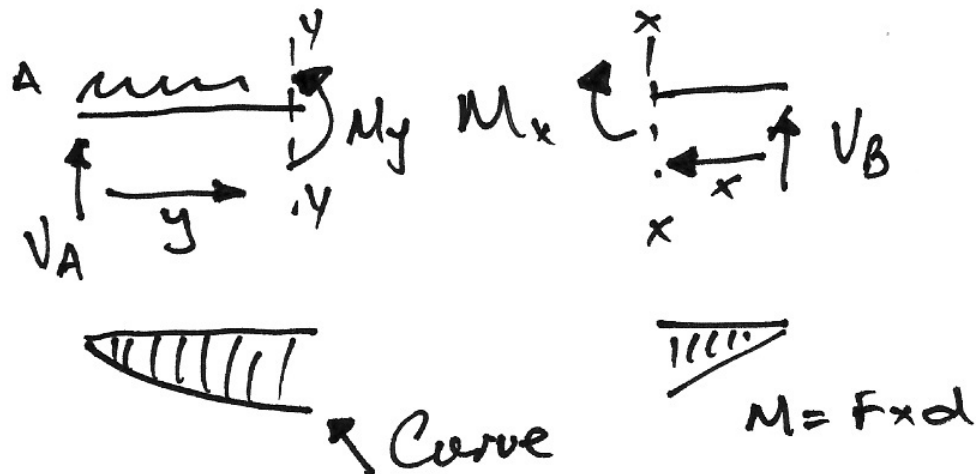
Solution

Using the same techniques as outlined in Example 1, we can quickly arrive at the deflected shape and reactions:

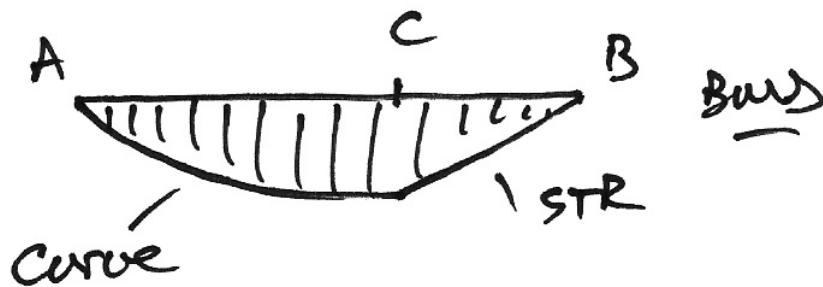


Based on the reactions, we can then examine the two portions of the structure for bending moments:

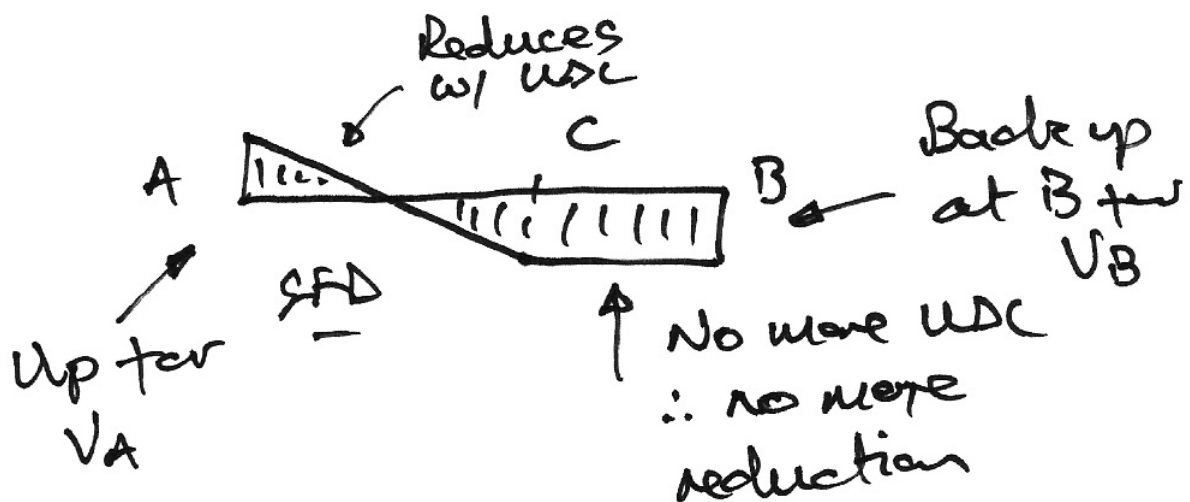
- *AC*: For this portion, the moment comes mainly from the reaction V_A . However, the moment gets progressively smaller than it would have been if just V_A was acting (i.e. force \times distance) since the UDL acts in the opposite direction. This means the BMD curves as shown below.
- *CB*: This portion is as studied in Example 1 is found from force \times distance.



Again, just like in Example 1, we recognize that we have no steps in the BMD and so join the moment diagrams for the two portions at C to get:



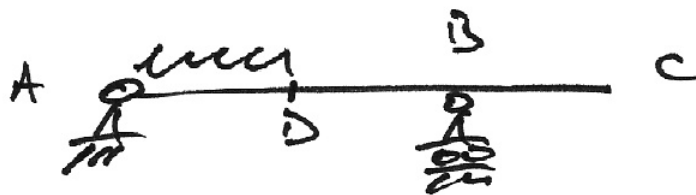
The shear force diagram is explained in the diagram:



2.6 Example 3

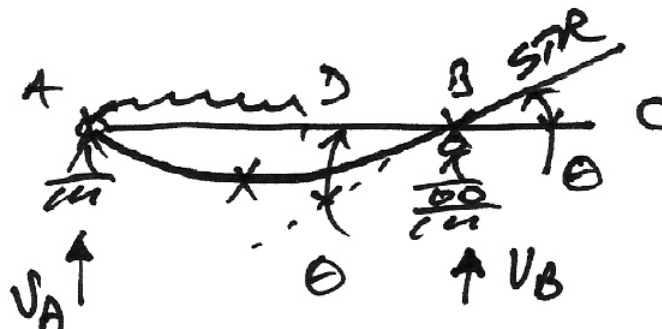
Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.



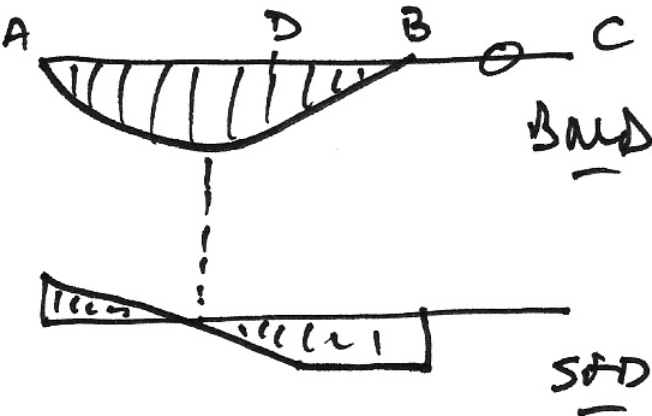
Solution

Again applying the techniques of Example 1 give the following deflected shape and reactions:



Note that for portion BC we recognize that there is no bending of the member. However, this does not mean that the member does not move: it does, and keeps a straight line extending the tangent to the deflected curve just to the left of B .

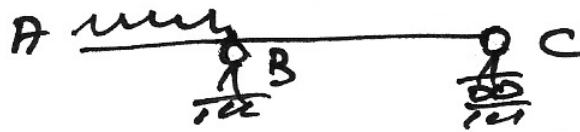
The bending moment and shear force diagrams are then found as per Example 2:



2.7 Example 4

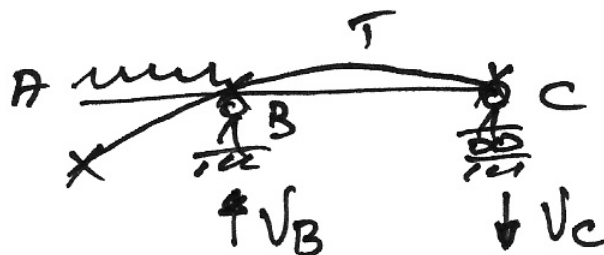
Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.



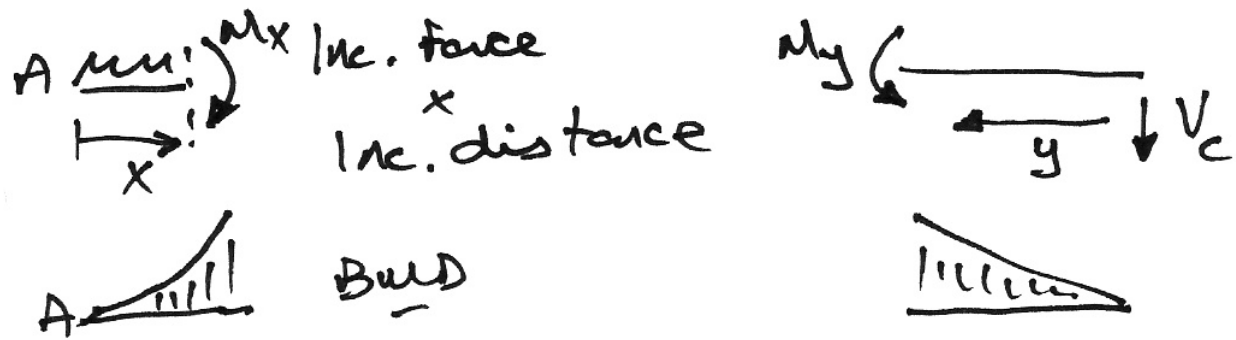
Solution

Again using points of certainty and removal of restraints we arrive at:

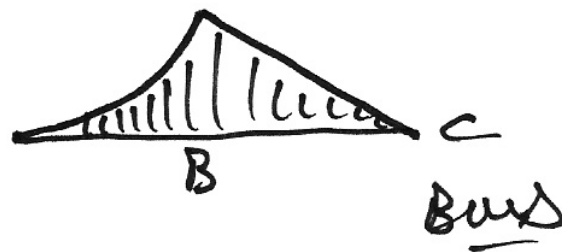


This allows us to look at the two portions of the structure for the BMD:

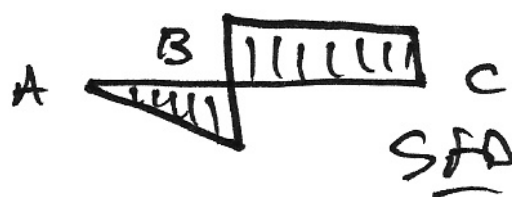
- *AB*: For this portion, we recognize that we have an increasing force (due to the accumulation of load from the UDL) as the distance increases. Thus we have a doubly increasing moment as the distance changes and so the BMD curves upwards as shown.
- *BC*: For this section, just apply force \times distance.



The BMD must not step (no applied moment) and so joins at B to yield:



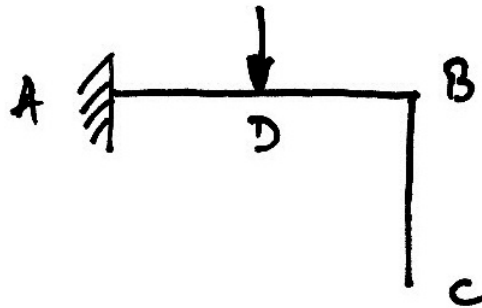
And the shear force diagram follows, either from the load types, or by looking at the BMD (curve to slope, line to constant).



2.8 Example 5

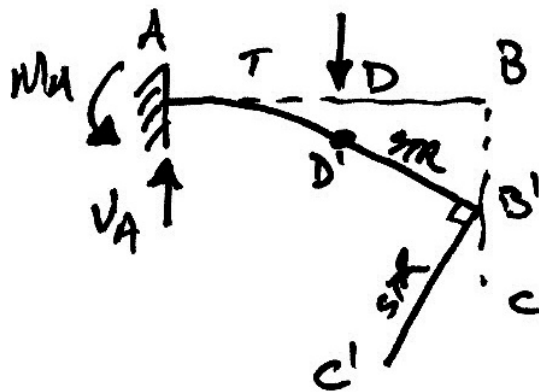
Problem

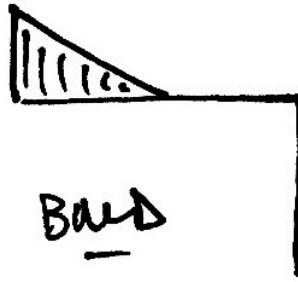
Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.



Solution

For this frame, we will start by establishing the reactions. First, since there is no

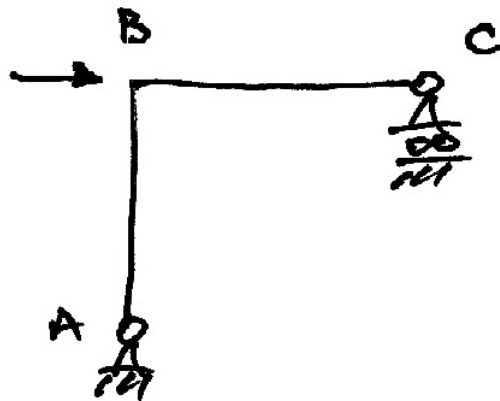




2.9 Example 6

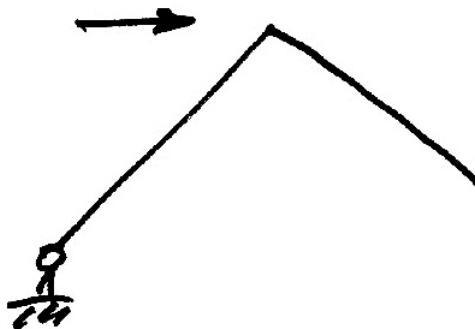
Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

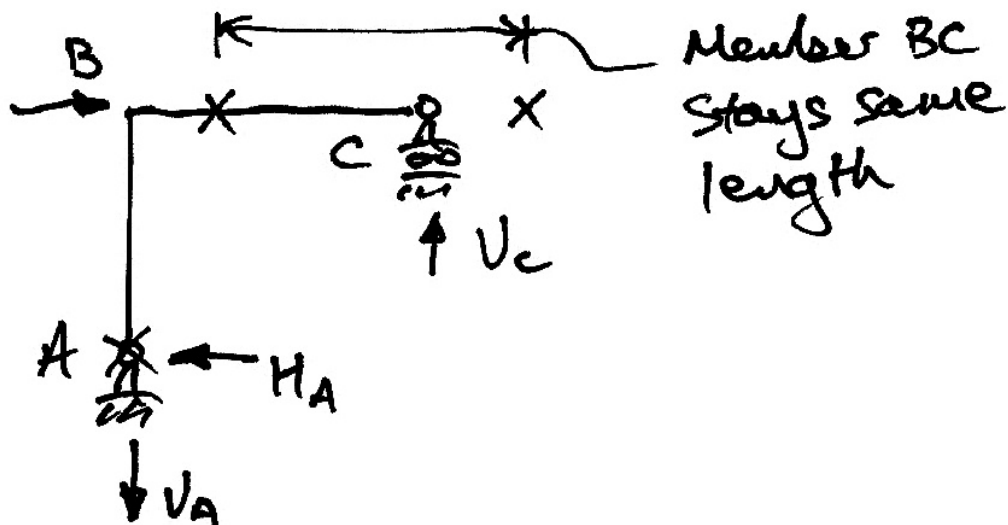


Solution

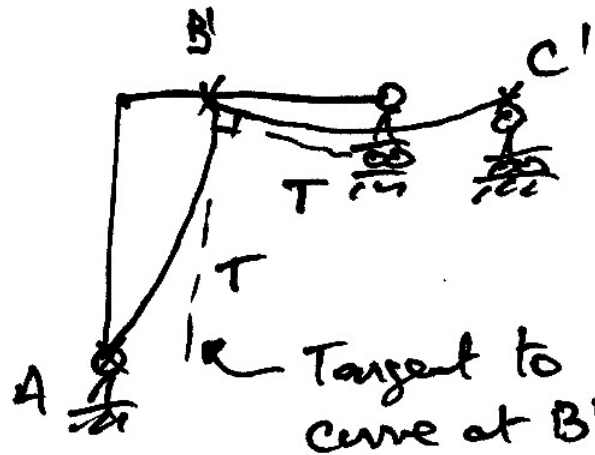
To begin, we will determine the direction of the vertical reaction at C. As ever, to do this, we will remove the restraint and examine what happens to the structure under the applied load:



Given this deflected shape, it is obvious that the vertical reaction at C should be upwards to keep C at the correct height. Since there are no vertical loads, this means that because of $\sum F_Y = 0$, we must have V_A acting downwards. Since there is no other possible horizontal force, by $\sum F_X = 0$ we have H_A acting to the left. Thus we have:

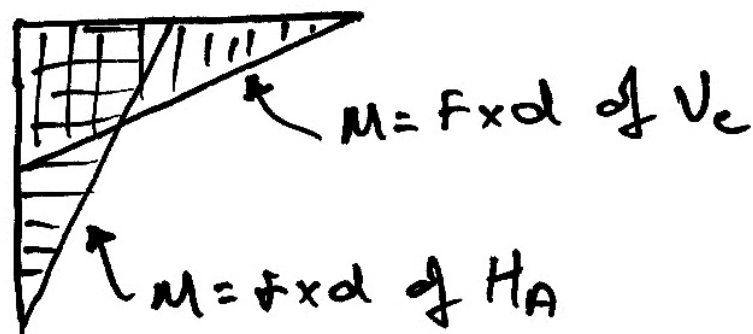


In the above diagram we have also indicated some points of certainty. That of A is easy due to the support. However, at B we note that the frame should move away from the load, but cannot move vertically downwards since member AB does not change length (ignoring axial deformation). This locates the deflected position of joint B . And, as indicated in the diagram, once the deflected location of joint B is known, so is that of joint C , because we know that member BC does not change length. Finally then, to assist us drawing the deflected shape between these points of certainty, we recognize that the joint is opening and so is rotating clockwise to give:

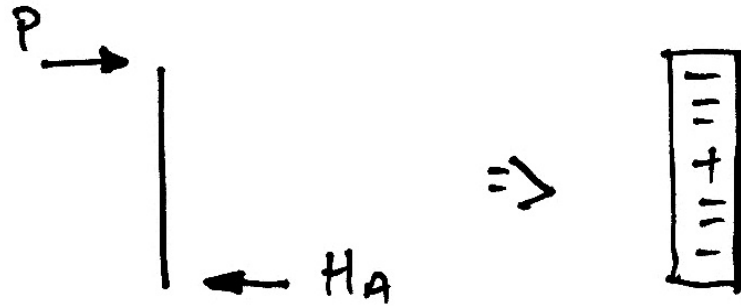


In the above, the tangents to the deflected shape curves are shown at joint B to demonstrate that the joint is rotating, but keeping its angle the same at B .

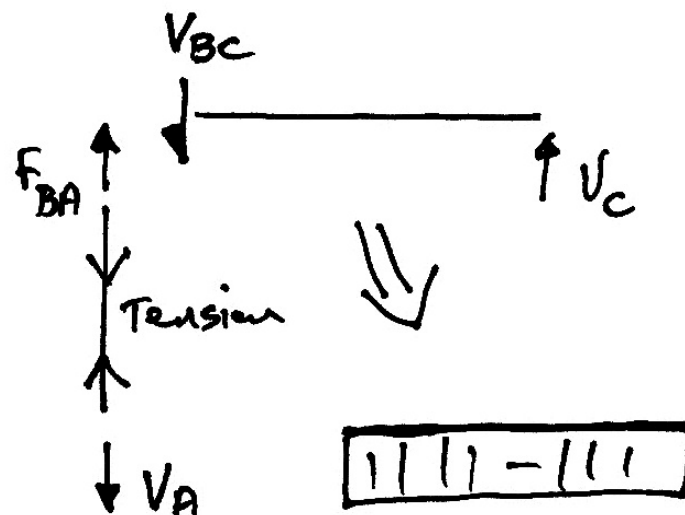
The BMD is easily established considering free-body diagrams of each member, along with the simple force \times distance:



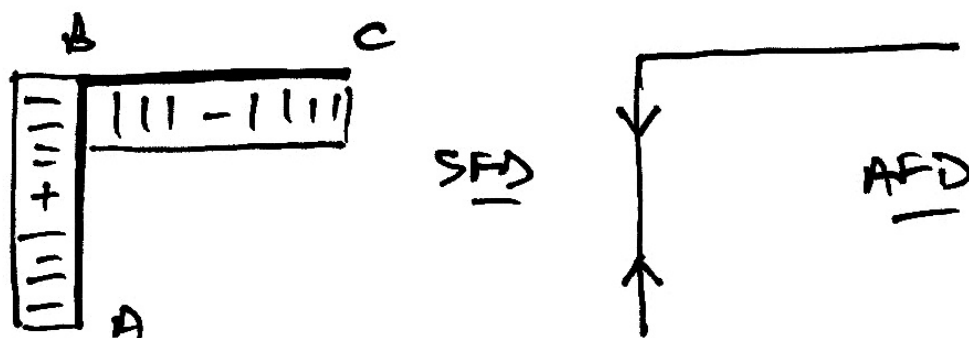
Recalling that shear is transverse forces to the member line, in considering the shear of member AB , we need only consider the applied load and H_A to get:



For the shear in member BC , we must first consider that the transverse force (besides V_C) gets there through member AB as an axial force (caused by V_A), to get:



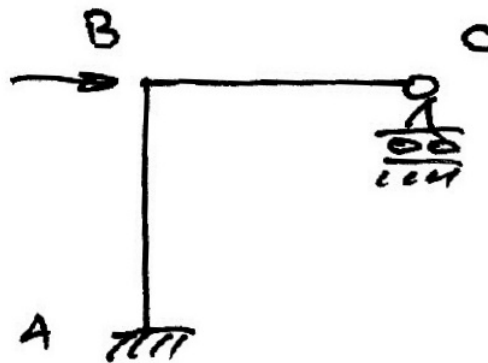
We combine the two solutions above to get the final shear and axial force diagrams:



2.10 Example 7

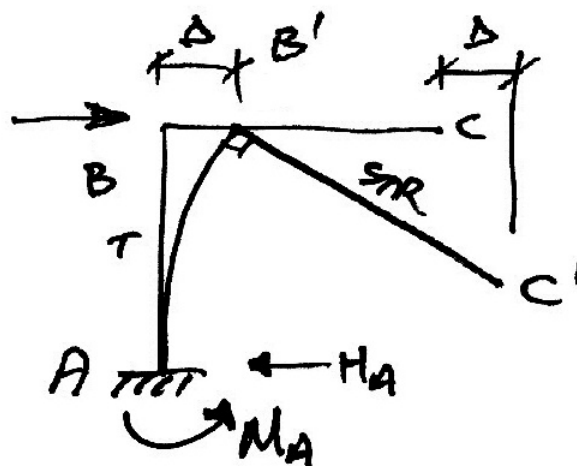
Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.



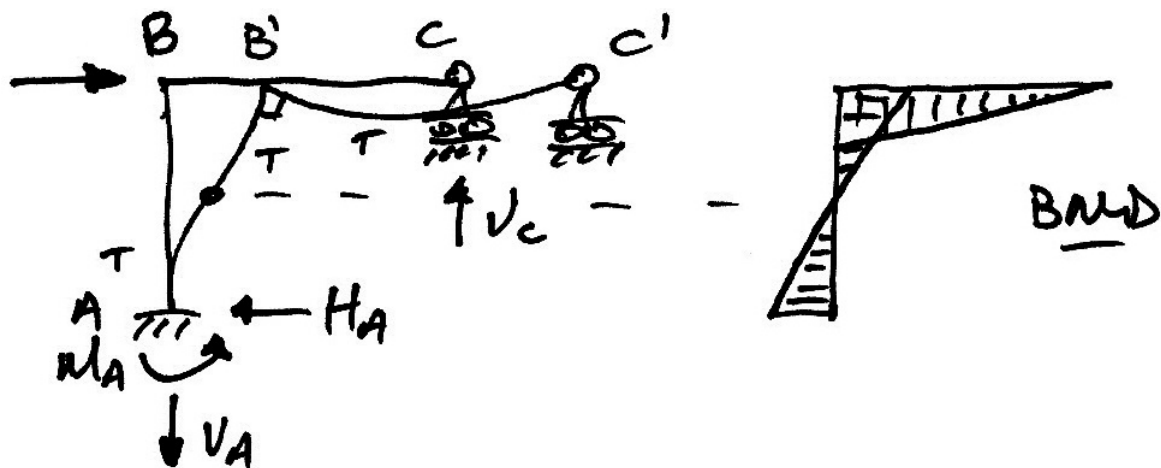
Solution

This frame is the same as that of Example 6, except for the support type at A. Thus we will see the influence of fixing a support on a structure. Firstly, proceed as we did before, and remove the support at C to get:



Notice that the diagram emphasises that the horizontal displacements at B and C must be the same since member BC does not change length. Also we see that we develop an anti-clockwise moment reaction at A .

Next, introduce the vertical support at C , noting that we now have an upwards vertical reaction at C , and proceed as before to get:



Notice that the *Point of Contraflexure* is noted as a dot in the deflected shape drawing, and its location is produced across to locate the zero point of bending moment on the column AB .

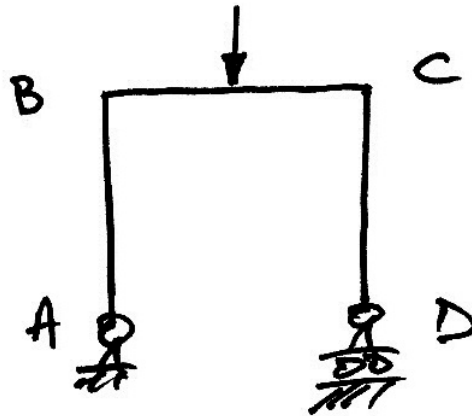
The shear force and axial force diagrams are obtained as was done in Example 6:



2.11 Example 8

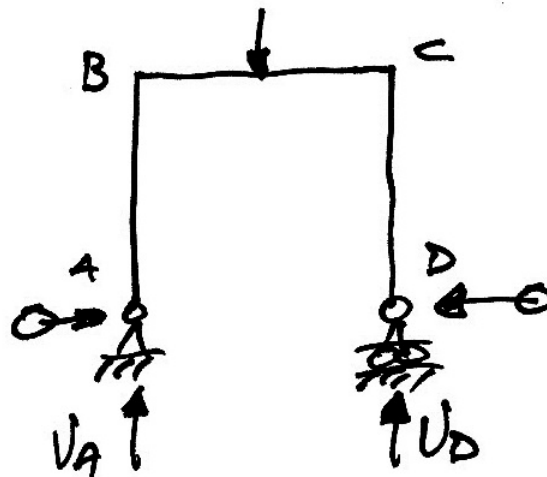
Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

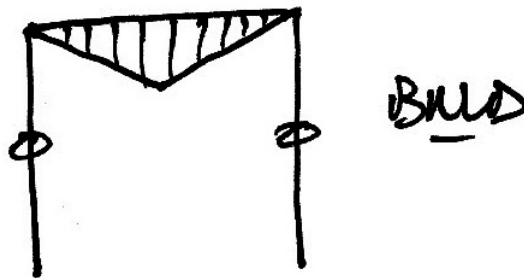


Solution

For this frame, we will start by establishing the reactions. First, since there is no horizontal support at D , and since $\sum F_x = 0$, we know $H_A = 0$. Also by considering removal of restraints we will see that the two vertical reactions are upwards, to give:



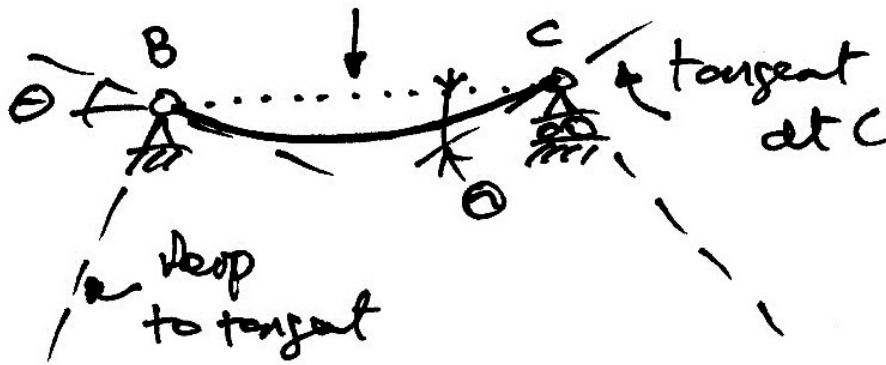
Bending moments are only caused by forces transverse to a member. Thus, with no horizontal reactions (i.e. no forces transverse to members AB or CD), there can be no bending moments in the columns. This only leaves the beam BC to act as a simply supported beam, giving the BMD as:



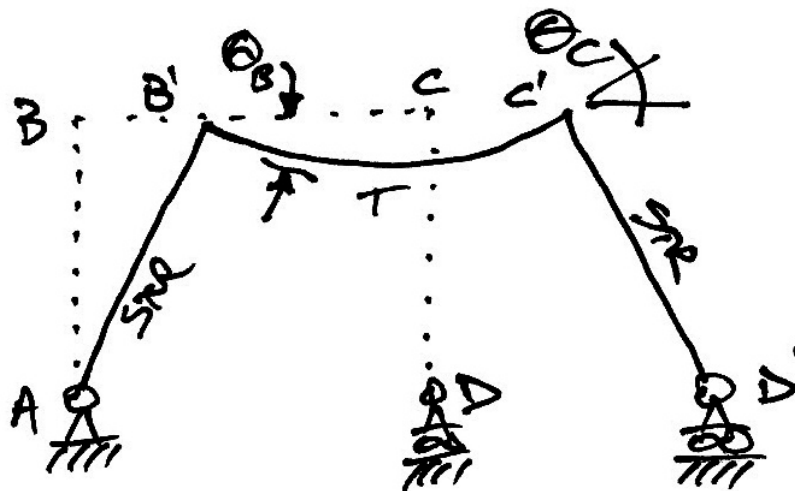
Next we note that the columns are in compression (by the reactions) and transmit the end shears of member BC to ground, whilst there is no axial force in the beam since there are no horizontal forces. The shear force and axial force diagrams are thus:



Lastly, we come to draw the deflected shape of this frame. However, before we do so, we recall that there is no bending in the columns and that member BC behaves as if a simply-supported beam. We examine the bending of member BC in more detail:



In this diagram we have identified the tangents to the end rotations of beam BC and the perpendiculars to these tangents. We recall that the right-angle rigid joints of the frame remain at right-angles, and so joints B and C of the frame rotate through θ . However, since there is no bending in member AB , and since A cannot move (pin support), B must move to B' so that the rotation θ can occur at B . Joint C and member CD behave similarly. Finally, we note that the distance BB' and CC' must be the same since member BC does not change length. All of this gives:

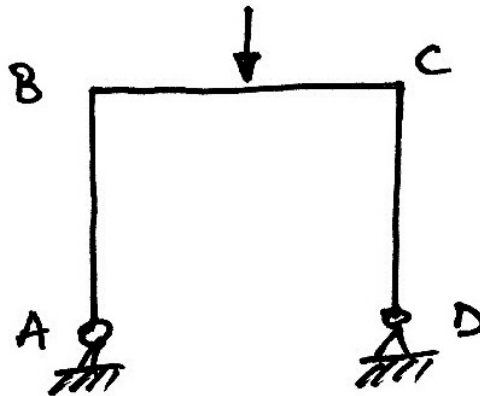


A way to think about it is that the frame sways to the right in order to avoid bending the columns.

2.12 Example 9

Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.



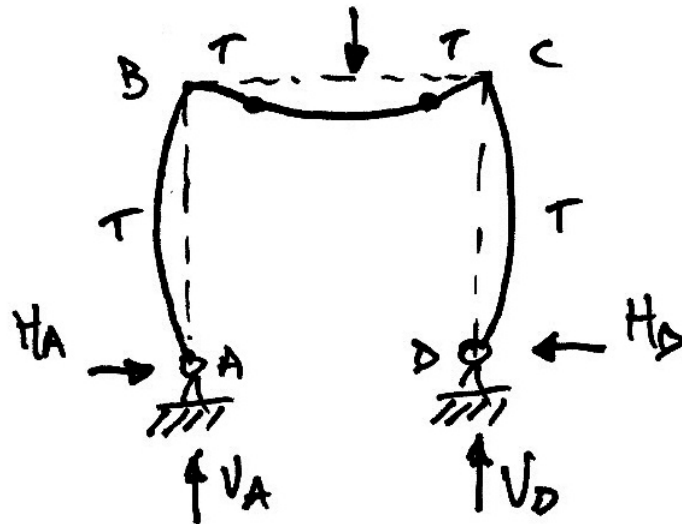
Solution

This frame is quite similar to the previous frame, except that D cannot move horizontally. This being the case, we must have a horizontal reaction acting to the left at D . Further, since $\sum F_x = 0$ we must have a horizontal reaction at A opposing H_D .

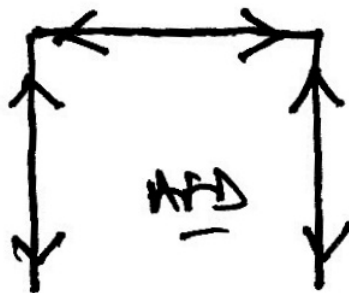
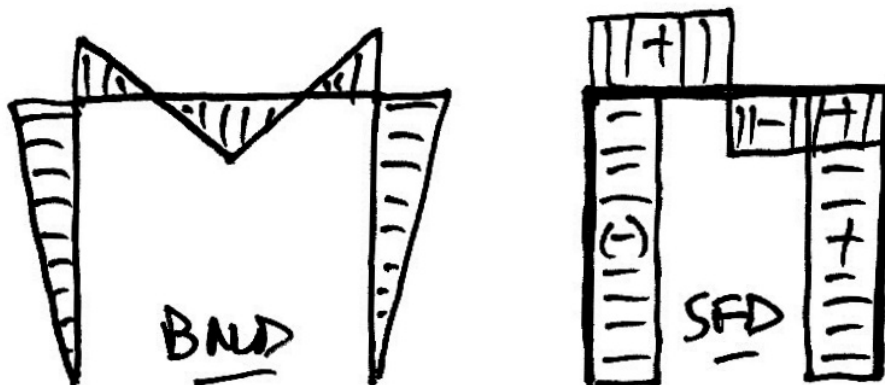
Please note this as it is a common misconception:

Just because there are no applied horizontal forces, does not mean there cannot be any horizontal reactions (but if there are, they must balance).

Finally for the reactions then, we note that the vertical supports must offer upwards reactions. Thus our deflected shape and reactions are:



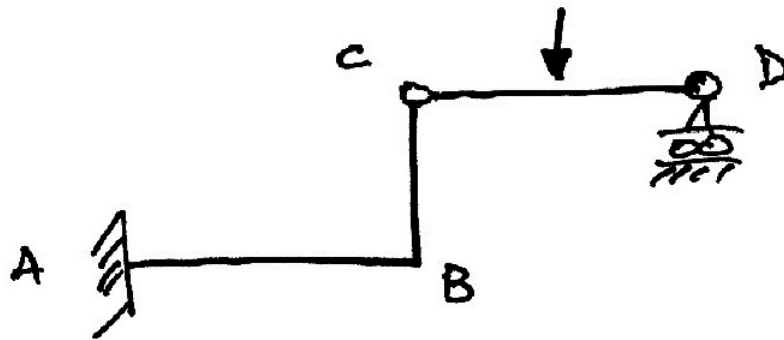
The BMD, SFD, and AFD follow directly by applying the techniques covered earlier given the reactions. Note especially that joints B and C are effectively closing and that beam BC behaves similar to a fixed-fixed beam:



2.13 Example 10

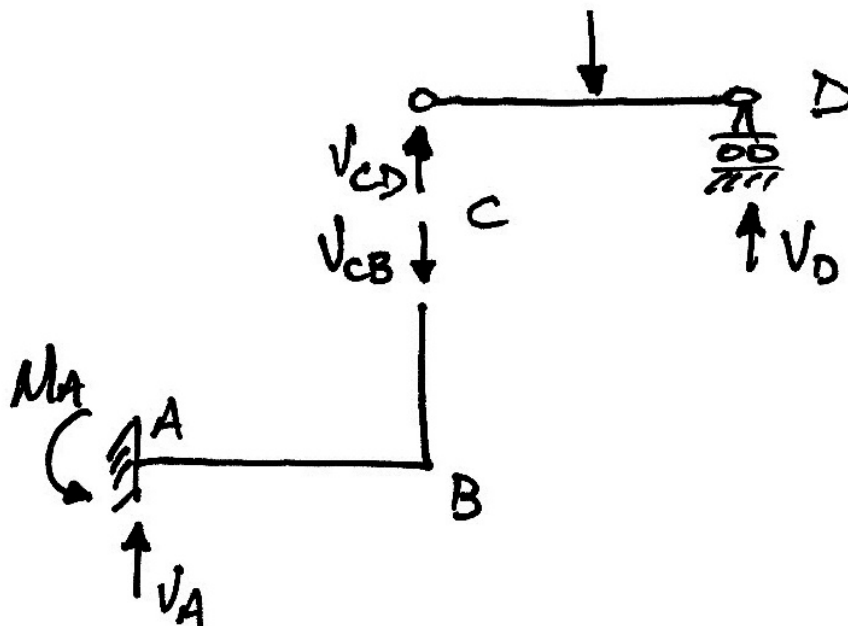
Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.



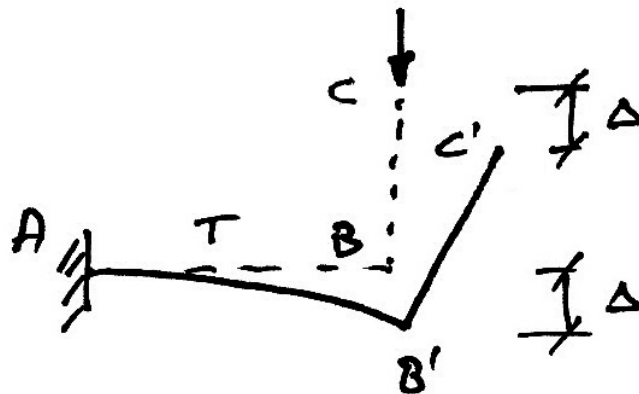
Solution

To proceed with this frame we will split it at the hinge:

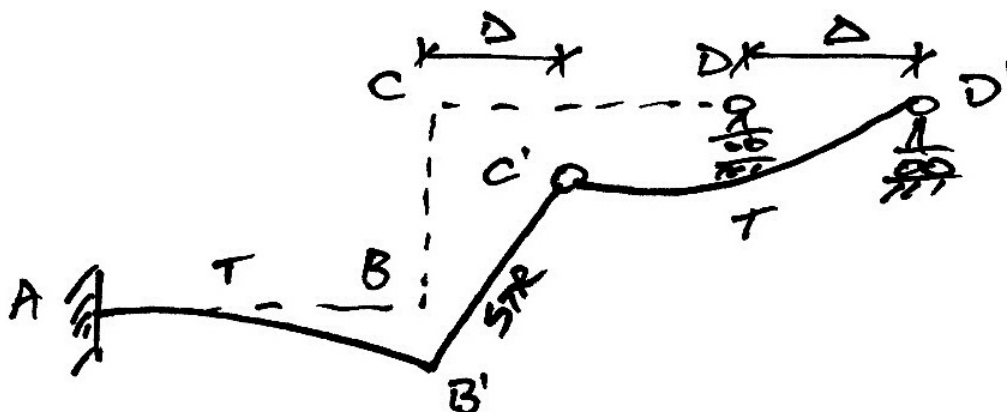


We note then that CD is effectively a simply supported beam, and this gives the interaction force direction as upwards for CD (reflecting the support that the structure ABC offers) and downwards for ABC (reflecting the push coming from the load on the beam). From V_{CB} we can determine the moment and vertical reactions at A .

The deflection behaviour of the beam CD is straightforward. We examine the deflection behaviour of ABC noting that B moves away from the load (downwards) and member BC maintains the perpendicular angle to the tangent at B . Moreover, member BC has no transverse force as so remains straight (i.e. does not bend). Lastly, see that the vertical movement of joints B and C must be the same since member BC does not change length:

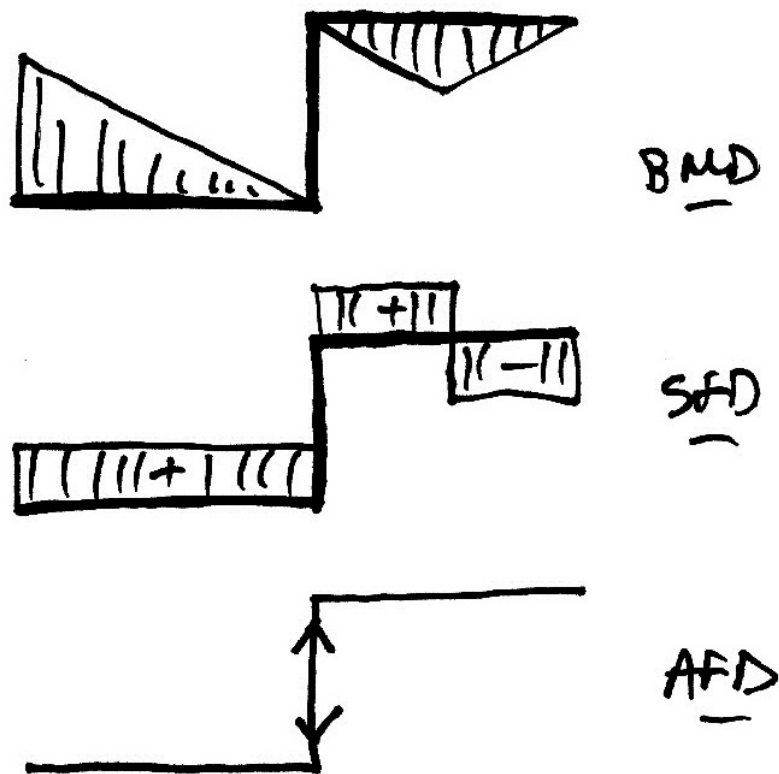


To obtain the overall deflected shape, add the above to that of the beam CD to get:



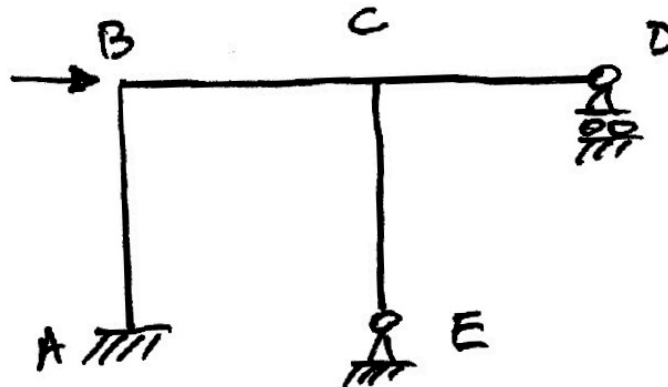
The diagram emphasises the point that the horizontal movement at C and D must be equal since the beam CD does not change length.

With the reactions and deflected shape established, the remaining diagrams follow easily using the techniques previously described:

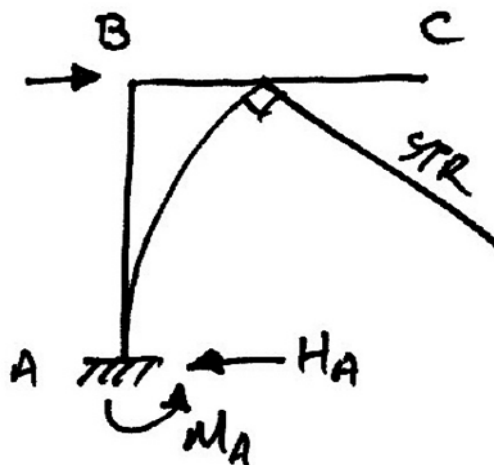


2.14 Example 11**Problem**

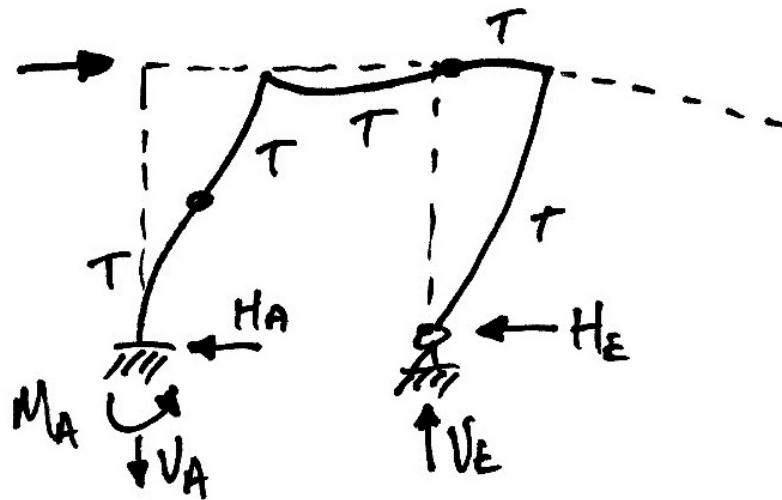
Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

**Solution**

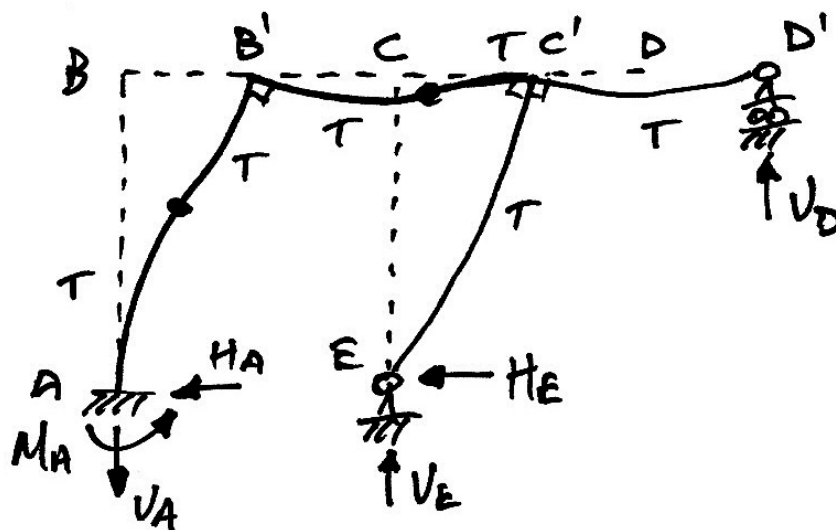
This is a more complex frame than previous frames, and so we will begin by cutting the structure back and gradually adding in the extra members. This is a bigger-scale removal of restraints method, where the members are considered as a type of restraint. We start with the portion ABC, which has been studied previously:



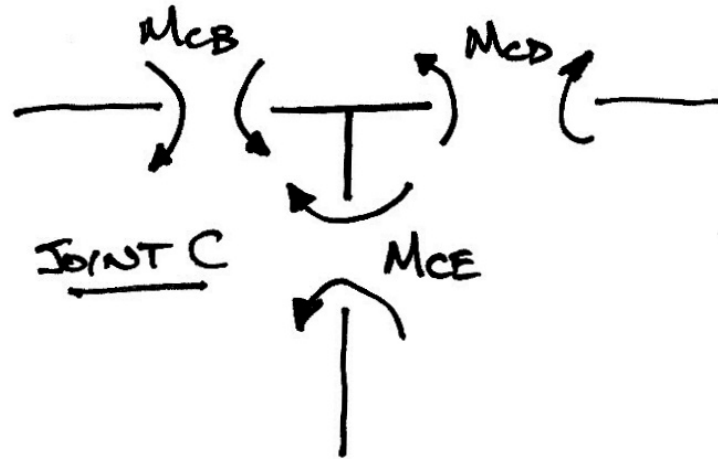
If we now introduce member CE , we can see that it must push upwards on joint C to keep C at the horizontal level it should be at (since member CE doesn't change length). This tells us that we have an upwards vertical reaction at E . And since $\sum F_y = 0$, we therefore know that V_A is downwards. Also there must be a horizontal reaction at E to keep E from moving right. This causes tension on the outside of member CE . All this is summarized in the following diagram.



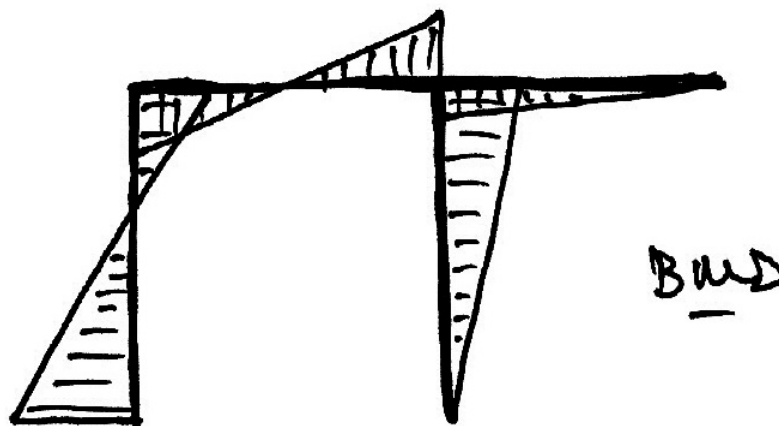
Notice that we have dotted in where member CD would be if it were connected. This tells us that the vertical reaction at D must be upwards as follows:



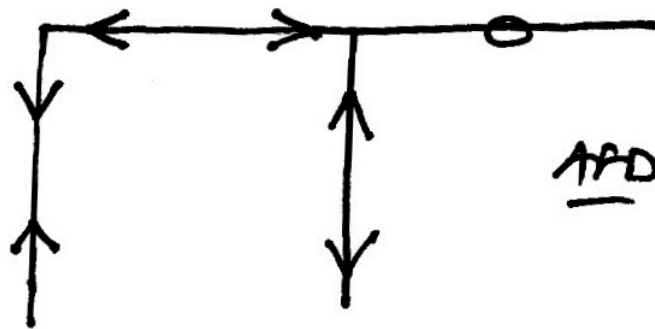
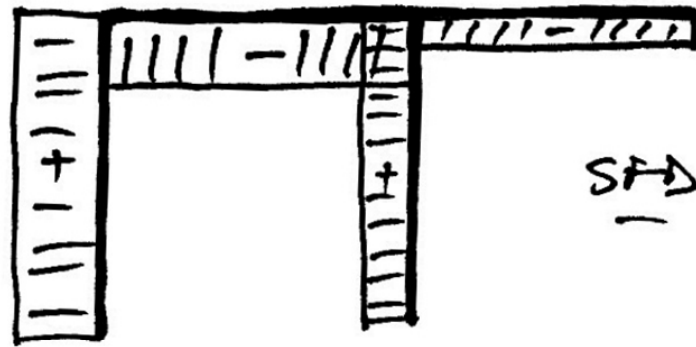
Since we know the sides of the members upon which there is tension, we can assess the equilibrium of joint C :



From this, we see that the bending moment in member CE is biggest. With this information, and the simple force \times distance strategy of earlier examples, we get:



The shear force and axial force diagrams follow similarly by considering the forces along or transverse to each member. One particularly notable point is that the applied horizontal load 'splits' at B : some goes through shear down to A (giving H_A), whilst the rest (probably smaller) puts member BC into compression before travelling down member CE in shear to give H_E . Thus we have:



3. Problems

3.1 *Introduction*

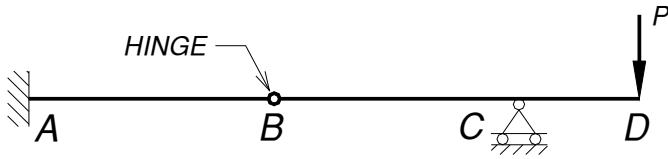
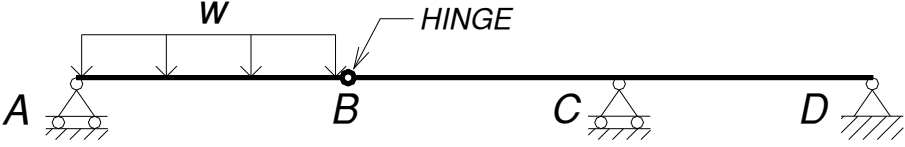
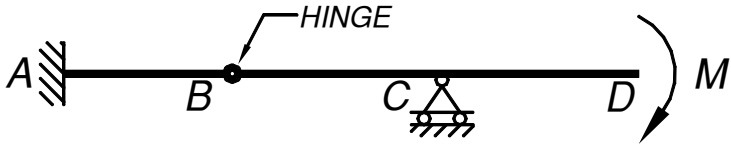
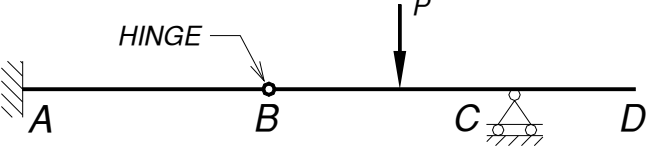
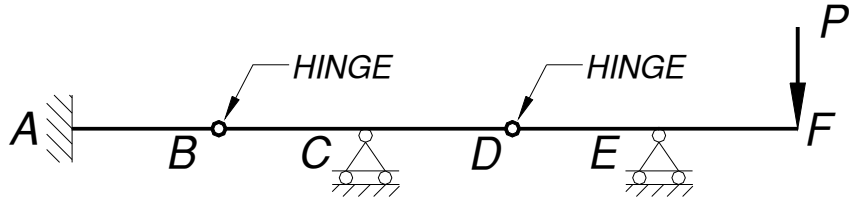
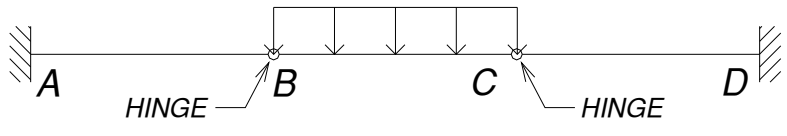
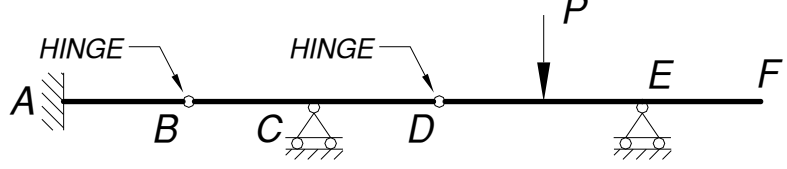
There is no better way to learn qualitative analysis than by practice. So here follows a good variety of determinate and indeterminate structures for analysis.

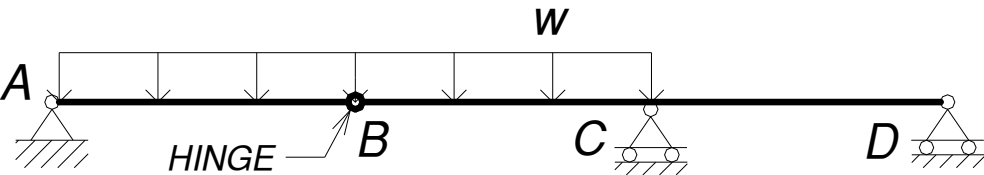
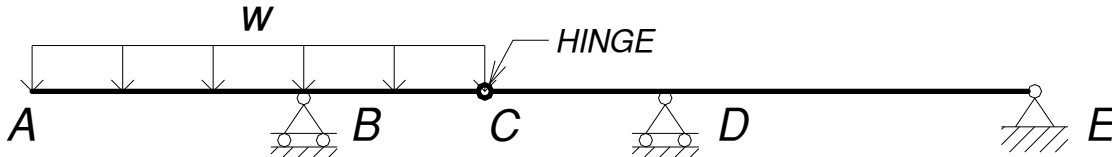
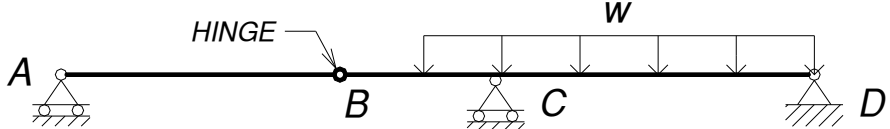
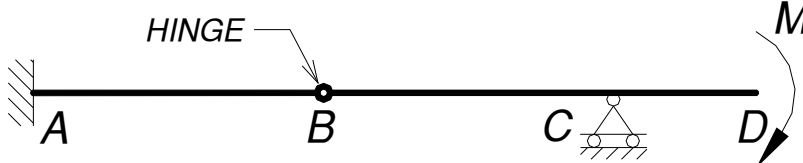
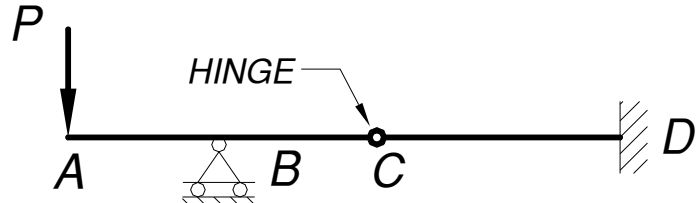
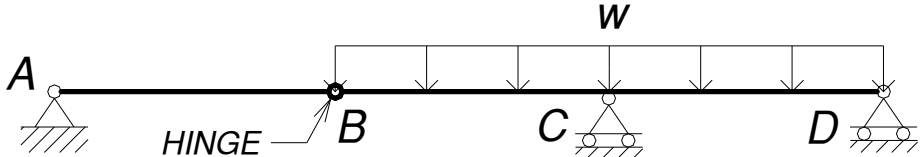
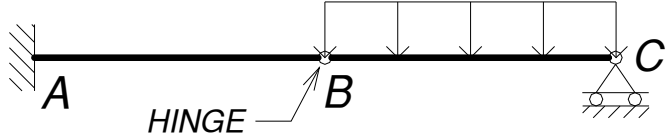
For each of the following structures, determine the:

- Reactions;
- Bending moment diagram;
- Shear force diagram;
- Axial force diagram;
- Deflected shape.

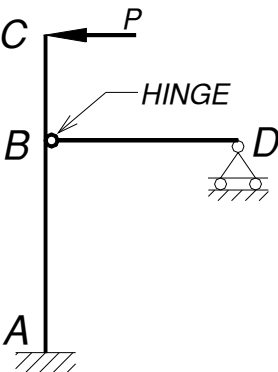
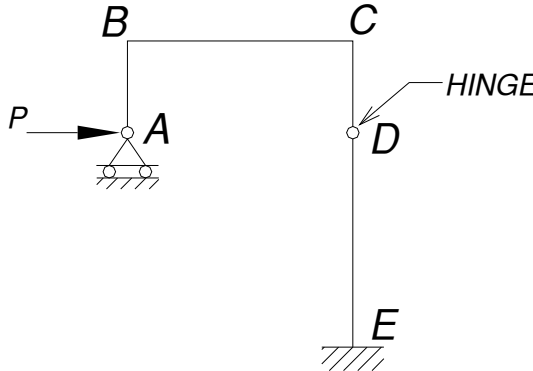
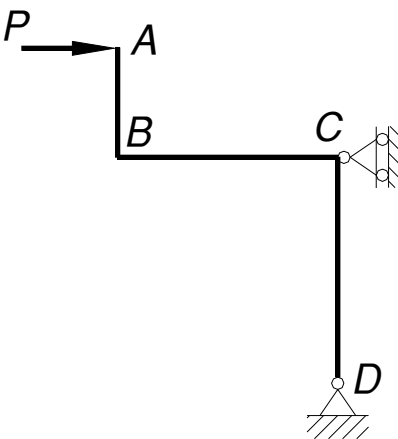
For the trusses, identify the sense of the force, if any, in each member.

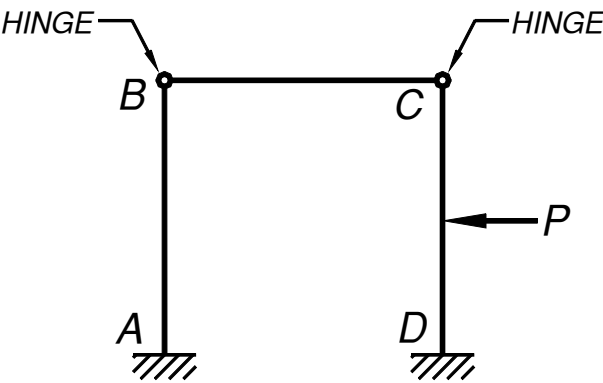
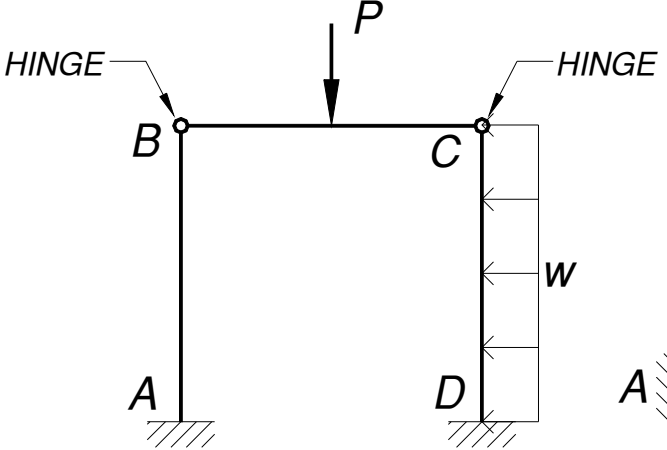
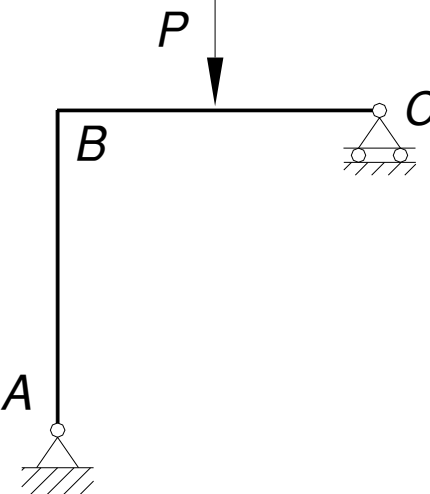
3.2 Statically Determinate Beams

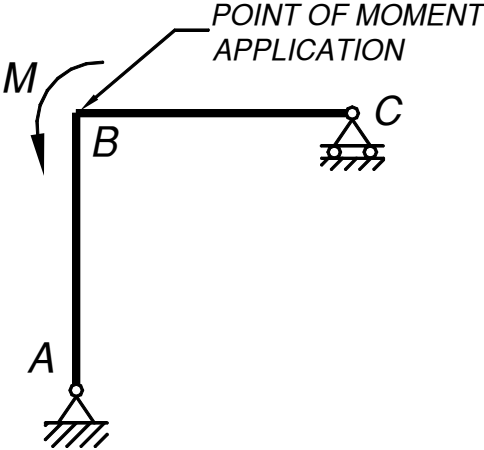
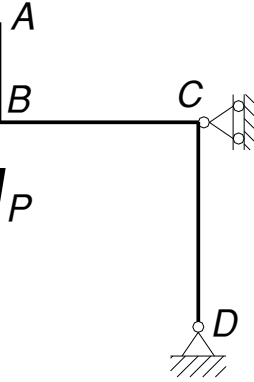
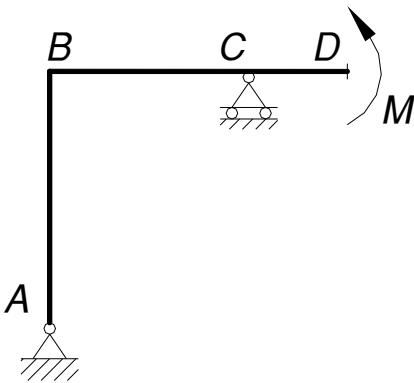
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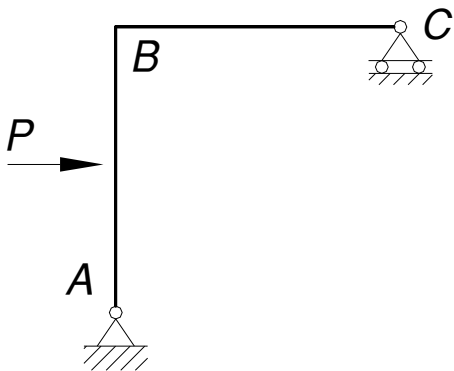
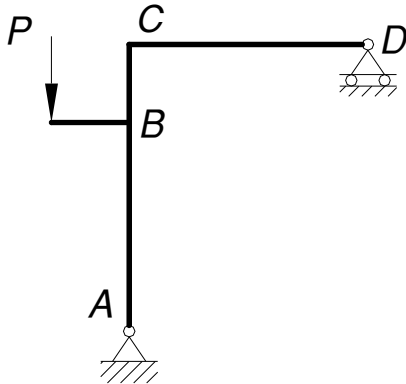
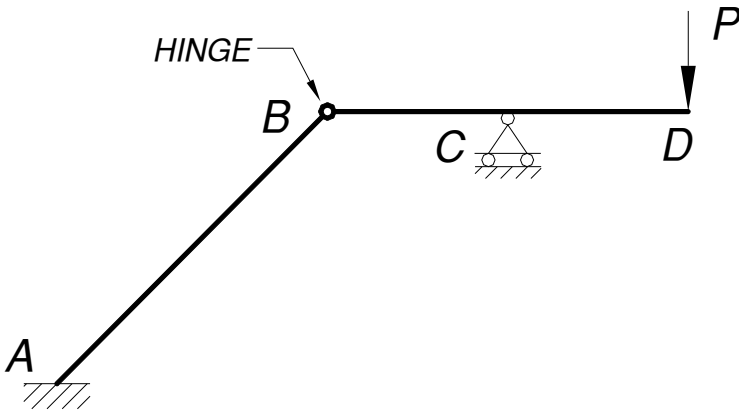
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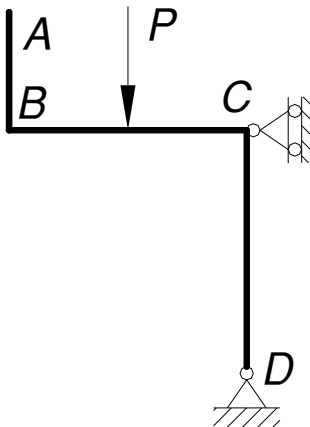
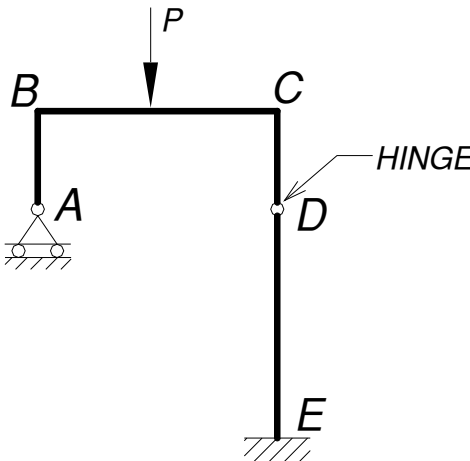
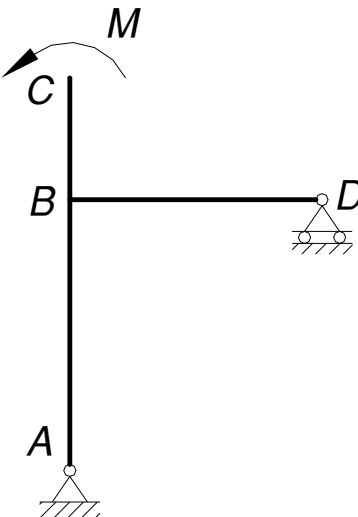
3.3 Statically Determinate Frames

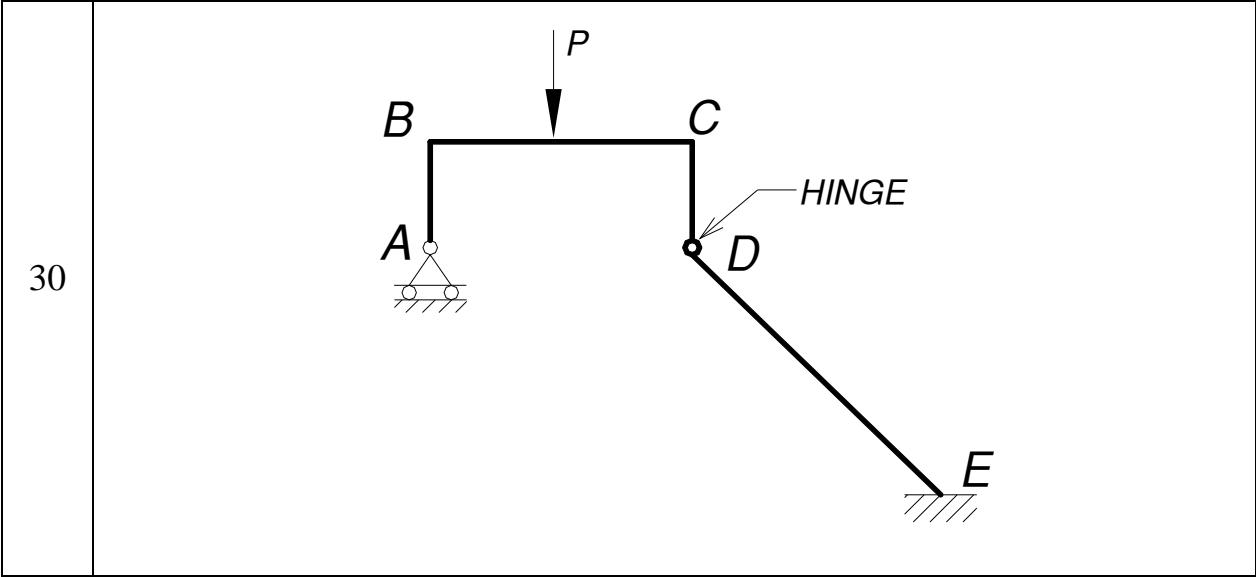
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18	 <p>Diagram of a rectangular frame structure. The frame consists of four joints: A (bottom-left), B (top-left), C (top-right), and D (bottom-right). Joints A and D are fixed supports, indicated by hatched lines. Joints B and C are hinges, indicated by circles and the label "HINGE". A horizontal force P is applied to the right at joint C.</p>
19	 <p>Diagram of a rectangular frame structure. The frame consists of four joints: A (bottom-left), B (top-left), C (top-right), and D (bottom-right). Joints A and D are fixed supports, indicated by hatched lines. Joints B and C are hinges, indicated by circles and the label "HINGE". A vertical force P is applied downwards at joint B. A horizontal force W is applied to the left at joint C.</p>
20	 <p>Diagram of an L-shaped frame structure. The frame consists of three joints: A (bottom-left), B (top-left), and C (top-right). Joint A is a pin support, indicated by a circle and hatched lines. Joint C is a roller support, indicated by a triangle and hatched lines. A vertical force P is applied downwards at joint B.</p>

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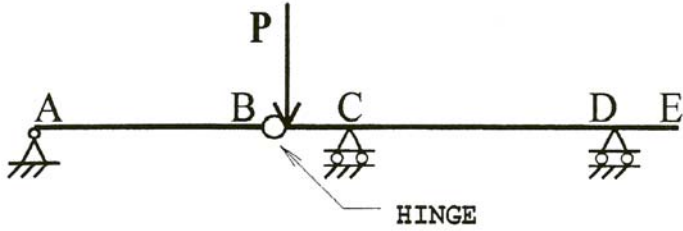
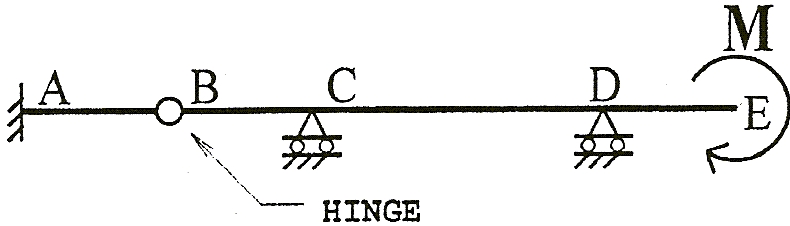
24	 <p>Diagram of a frame structure. It consists of a vertical member AB and a horizontal member BC. Support A is a pin support. Support C is a roller support. A horizontal force P is applied at joint B, pointing to the right.</p>
25	 <p>Diagram of a frame structure. It consists of a vertical member AB and a horizontal member CD. Support A is a pin support. Support D is a roller support. A horizontal force P is applied at joint B, pointing to the left.</p>
26	 <p>Diagram of a frame structure. It consists of a diagonal member AB and a horizontal member BD. Support A is a pin support. Support C is a roller support. A vertical force P is applied at joint D, pointing downwards. A hinge is located at joint B, as indicated by the label "HINGE" and an arrow pointing to joint B.</p>

27	 <p>Diagram of a frame structure. It consists of a vertical member AB, a horizontal member BC, and a vertical member CD. A downward point load P is applied at the midpoint of BC. Support A is a fixed support, and support D is a roller support.</p>
28	 <p>Diagram of a frame structure. It consists of a vertical member AB, a horizontal member BC, and a vertical member CE. A downward point load P is applied at the midpoint of BC. Support A is a roller support, and support E is a fixed support. A hinge is located at point D on member CE.</p>
29	 <p>Diagram of a frame structure. It consists of a vertical member AC, a horizontal member BD, and a vertical member DE. A clockwise moment M is applied at point C. Support A is a fixed support, and support D is a roller support.</p>

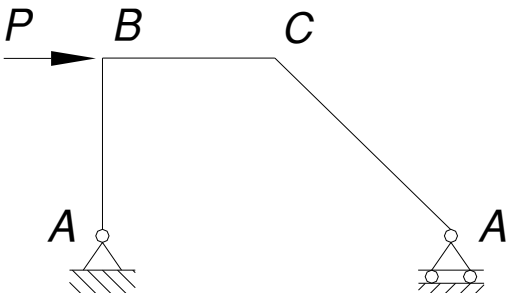
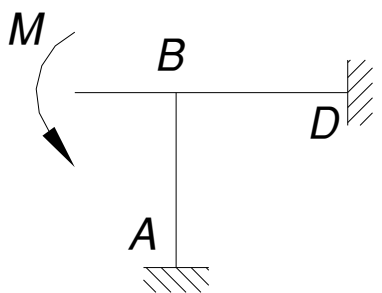
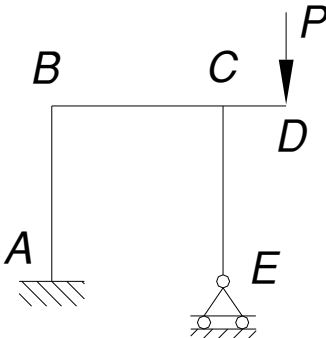
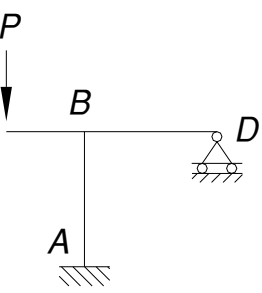


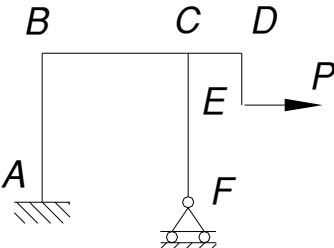
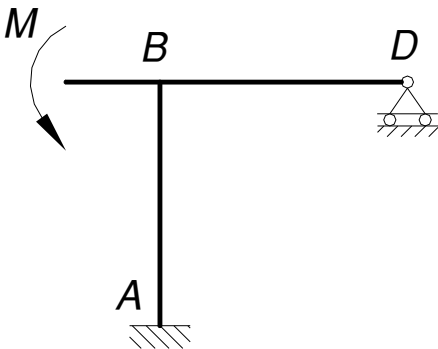
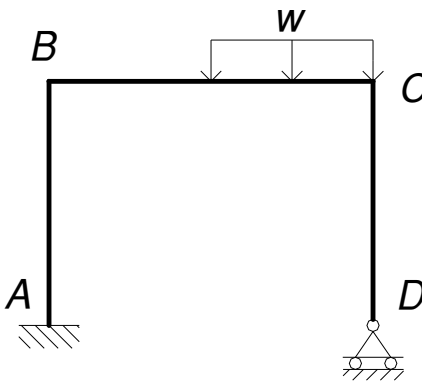
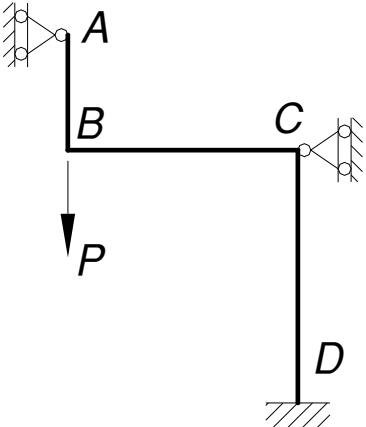
3.4 Statically Indeterminate Beams

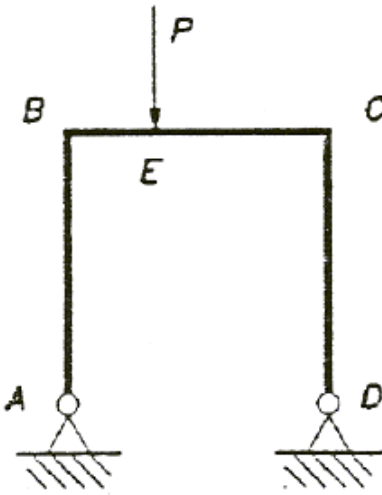
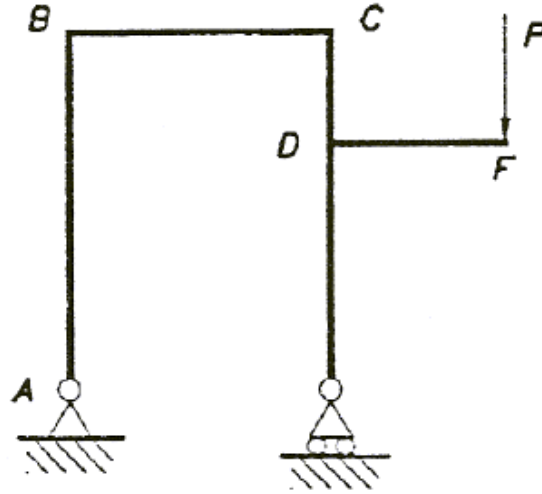
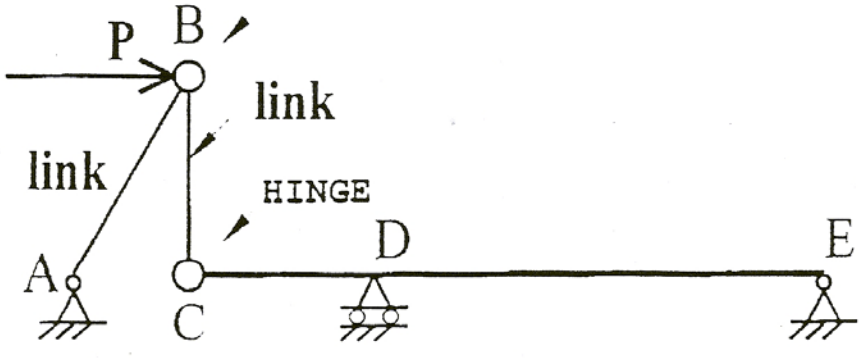
31	<p>Diagram of a beam with a fixed support at A, roller supports at B, C, and F, and an internal hinge at D. A downward point load P is applied at E.</p>
32	<p>Diagram of a beam with a fixed support at A, roller supports at C and F, and internal hinges at B, D, and E. A downward point load P is applied at D.</p>
33	<p>Diagram of a beam with a fixed support at A, roller supports at B and C, and a free end at D. A downward point load P is applied at D.</p>
34	<p>Diagram of a beam with a fixed support at A, roller supports at B, F, and G, and internal hinges at C and E. A downward point load P is applied at D.</p>
35	<p>Diagram of a beam with a fixed support at A, roller supports at C and D, and an internal hinge at B. A clockwise moment M is applied at E.</p>

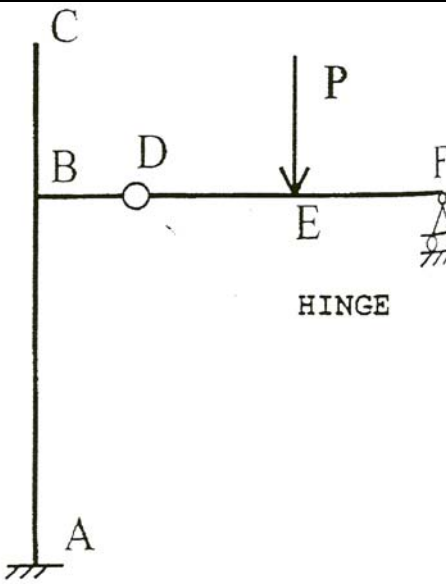
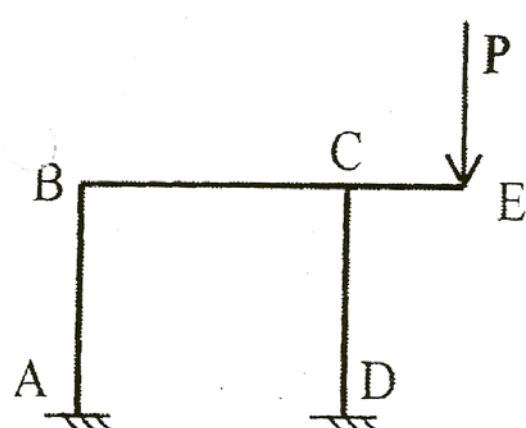
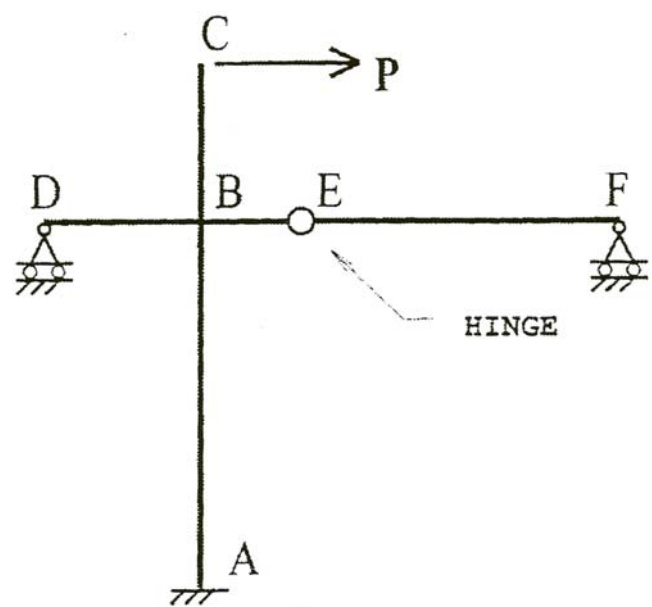
36	 <p>Diagram of a beam structure. The beam is horizontal and labeled with points A, B, C, D, and E from left to right. At point A, there is a pin support. At point B, there is a hinge and a downward point load labeled P. At point C, there is a roller support. At point D, there is a roller support. The end of the beam is at point E. An arrow points to the hinge at B with the label "HINGE".</p>
37	 <p>Diagram of a beam structure. The beam is horizontal and labeled with points A, B, C, D, and E from left to right. At point A, there is a fixed support. At point B, there is a hinge. At point C, there is a roller support. At point D, there is a roller support. At point E, there is a clockwise moment labeled M. An arrow points to the hinge at B with the label "HINGE".</p>

3.5 Statically Indeterminate Frames

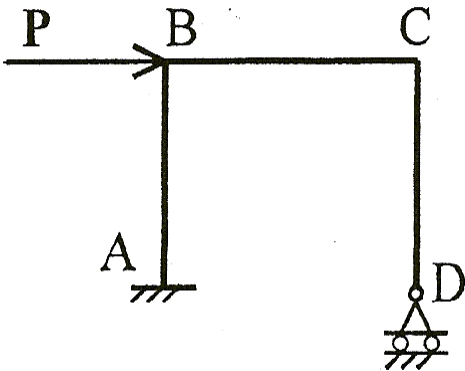
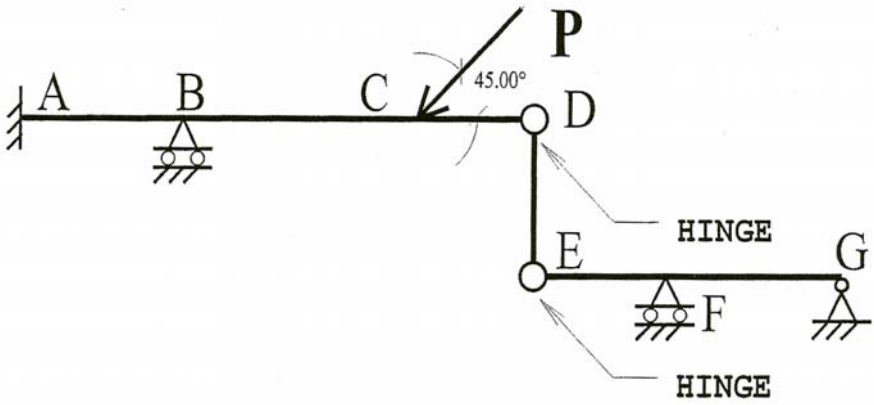
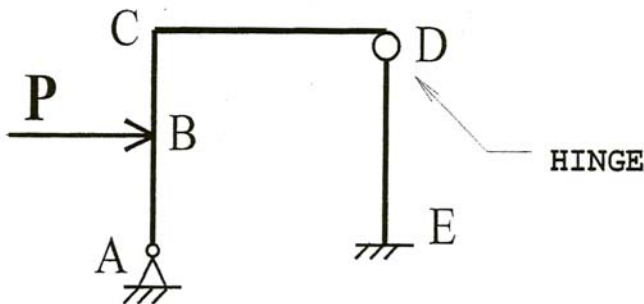
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46	 <p>A portal frame structure with two vertical columns and a horizontal beam. The left column is pinned at support A, and the right column is pinned at support D. The top of the left column is labeled B, and the top of the right column is labeled C. The horizontal beam is labeled E. A vertical downward load P is applied at the center of the beam E.</p>
47	 <p>A portal frame structure with two vertical columns and a horizontal beam. The left column is pinned at support A, and the right column is pinned at support D. The top of the left column is labeled B, and the top of the right column is labeled C. A horizontal member is attached to the right column at point D, extending to point F. A vertical downward load P is applied at point F. The horizontal member is labeled D.</p>
48	 <p>A frame structure with a horizontal beam and a vertical column. The left end of the horizontal beam is pinned at support A. The right end of the horizontal beam is pinned at support E. A vertical column is attached to the horizontal beam at point C. The top of the vertical column is labeled B. A horizontal load P is applied at point B. A hinge is located at point C. The horizontal beam is labeled D. The vertical column is labeled link. The horizontal beam is labeled link. The hinge is labeled HINGE.</p>

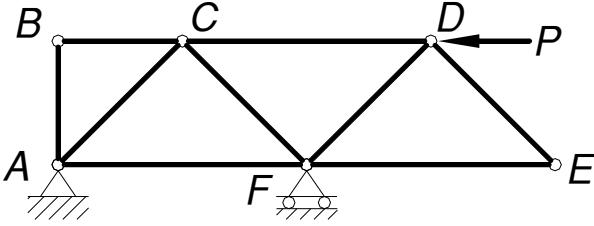
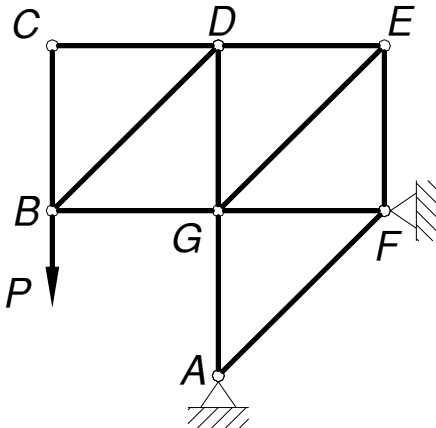
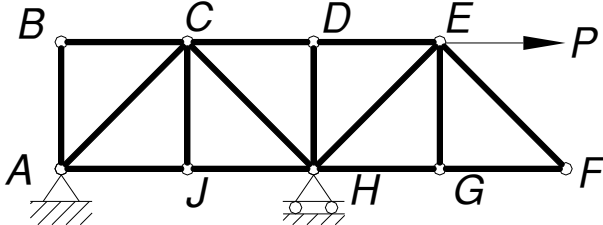
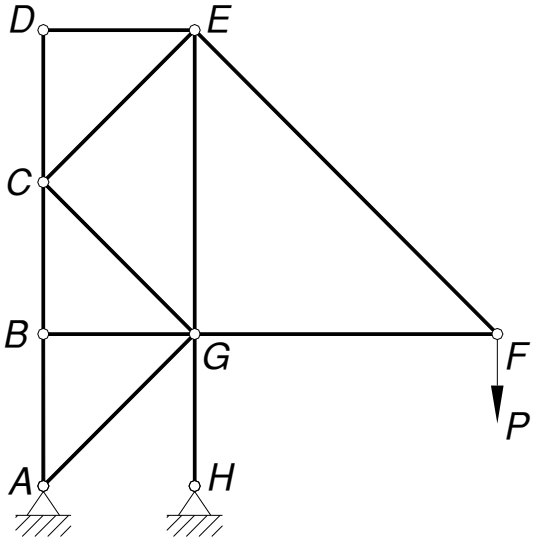
49	 <p>Diagram of a frame structure. It consists of a vertical member AB fixed at support A at the bottom. At joint B, a horizontal member BE extends to the right. A roller support is located at joint F, which is at the end of member BE. A hinge is located at joint D on member BE between joints B and E. A vertical downward load P is applied at joint E. The top of the vertical member is labeled C.</p>
50	 <p>Diagram of a frame structure. It consists of two vertical members AB and CD, both fixed at supports A and D at the bottom. They are connected by a horizontal member BC. At joint C, another horizontal member CE extends to the right, ending at a roller support at joint E. A vertical downward load P is applied at joint E.</p>
51	 <p>Diagram of a frame structure. It consists of a vertical member AB fixed at support A at the bottom. At joint B, a horizontal member DF extends to the right. There are roller supports at joints D and F. A hinge is located at joint E on member DF between joints B and F. A horizontal load P is applied at joint C, which is at the top of the vertical member AB, acting to the right.</p>

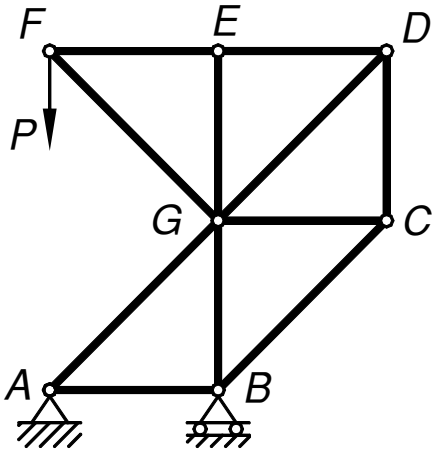
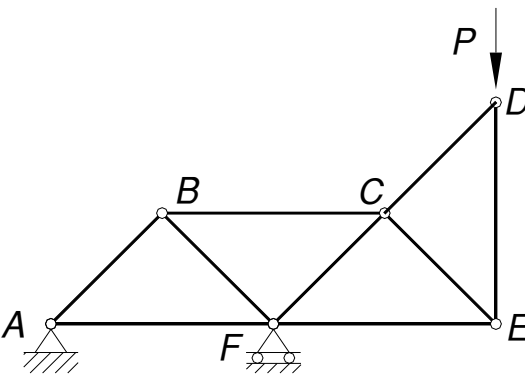
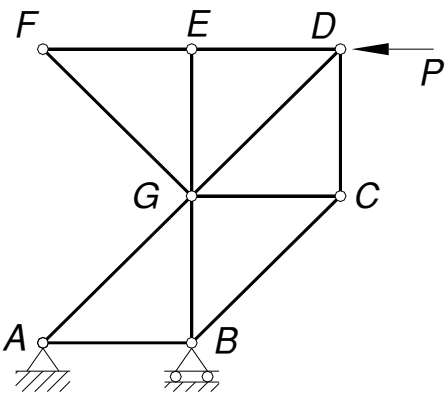
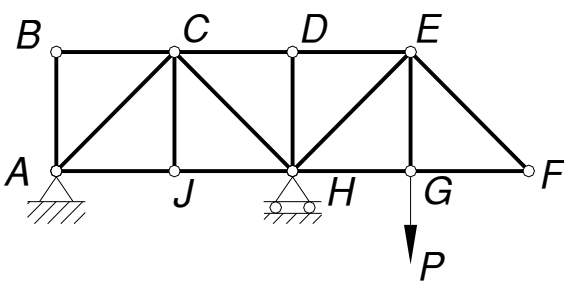
52	<p>Diagram of a frame structure. It consists of a vertical member AB fixed at support A, and a horizontal member BE. A hinge is located at point C on the horizontal member. A downward point load P is applied at point D on the horizontal member. The horizontal member is supported by a roller support at point E. A dashed arrow points to the hinge at C with the label "HINGE".</p>
53	<p>Diagram of a frame structure. It consists of a vertical member AB fixed at support A, a horizontal member BC, and a vertical member CE fixed at support E. A downward point load P is applied at point D on the horizontal member BC.</p>
54	<p>Diagram of a frame structure. It consists of a member AB fixed at support A, a horizontal member CD, and a diagonal member BC. A downward point load P is applied at point C. The horizontal member CD is supported by a roller support at point D.</p>

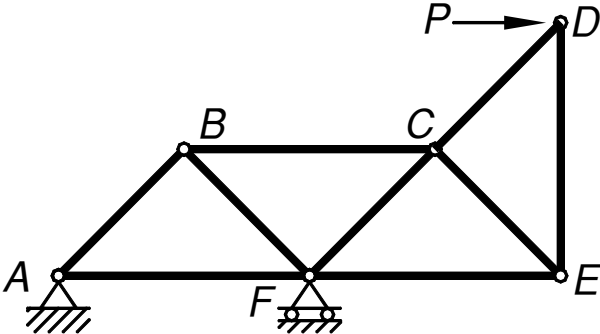
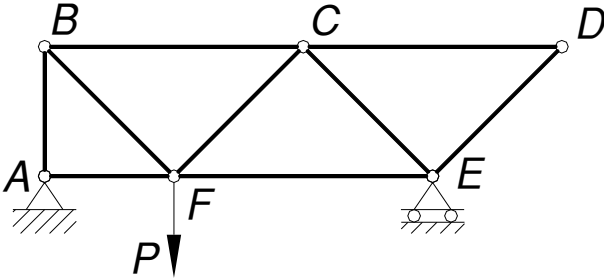
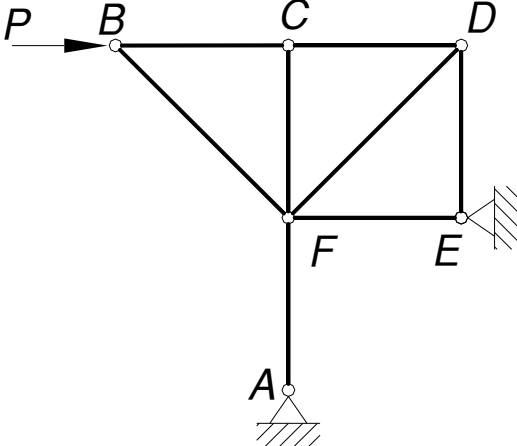
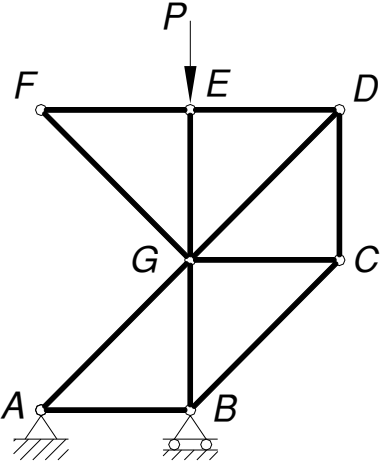
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3.6 Trusses

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66	 <p>Diagram of a truss structure with nodes A, B, C, D, E, F, and G. Node A is a pin support, and node B is a roller support. A downward force P is applied at node F. The truss consists of a rectangular frame ABCD with a diagonal member AC. A vertical member EG connects node E (top) to node G (middle), and another vertical member BG connects node B (bottom) to node G. A horizontal member FC connects node F (left) to node C (right). A diagonal member FD connects node F to node D. A horizontal member FE connects node F to node E. A vertical member ED connects node E to node D. A horizontal member GC connects node G to node C. A diagonal member CD connects node C to node D. A horizontal member AB connects node A to node B. A diagonal member AG connects node A to node G.</p>
67	 <p>Diagram of a truss structure with nodes A, B, C, D, E, and F. Node A is a pin support, and node F is a roller support. A downward force P is applied at node D. The truss consists of a rectangular frame ABCF with a diagonal member AC. A vertical member DE connects node D (top) to node E (bottom). A horizontal member BE connects node B (left) to node E (right). A diagonal member BD connects node B to node D. A horizontal member AF connects node A to node F. A diagonal member BF connects node B to node F. A horizontal member BC connects node B to node C. A diagonal member CE connects node C to node E. A horizontal member CD connects node C to node D. A vertical member DE connects node D to node E. A horizontal member FE connects node F to node E. A diagonal member FD connects node F to node D. A horizontal member AB connects node A to node B. A diagonal member AG connects node A to node G.</p>
68	 <p>Diagram of a truss structure with nodes A, B, C, D, E, F, and G. Node A is a pin support, and node B is a roller support. A horizontal force P is applied at node D, pointing to the left. The truss consists of a rectangular frame ABCD with a diagonal member AC. A vertical member EG connects node E (top) to node G (middle), and another vertical member BG connects node B (bottom) to node G. A horizontal member FC connects node F (left) to node C (right). A diagonal member FD connects node F to node D. A horizontal member FE connects node F to node E. A vertical member ED connects node E to node D. A horizontal member GC connects node G to node C. A diagonal member CD connects node C to node D. A horizontal member AB connects node A to node B. A diagonal member AG connects node A to node G.</p>
69	 <p>Diagram of a truss structure with nodes A, B, C, D, E, F, G, H, and J. Node A is a pin support, and node H is a roller support. A downward force P is applied at node G. The truss consists of a rectangular frame ABCD with a diagonal member AC. A vertical member DE connects node D (top) to node E (bottom). A horizontal member BE connects node B (left) to node E (right). A diagonal member BD connects node B to node D. A horizontal member AF connects node A to node F. A diagonal member BF connects node B to node F. A horizontal member BC connects node B to node C. A diagonal member CE connects node C to node E. A horizontal member CD connects node C to node D. A vertical member DE connects node D to node E. A horizontal member FE connects node F to node E. A diagonal member FD connects node F to node D. A horizontal member AB connects node A to node B. A diagonal member AG connects node A to node G.</p>

70	 <p>A truss structure with joints A, B, C, D, E, and F. Joint A is a pin support, and joint F is a roller support. The truss consists of a horizontal bottom chord A-F-E, a horizontal top chord B-C, and a vertical member D-E. Diagonal members connect A-B, B-F, F-C, and C-D. A horizontal force P is applied at joint D, pointing to the right.</p>
71	 <p>A truss structure with joints A, B, C, D, E, and F. Joint A is a pin support, and joint E is a roller support. The truss consists of a horizontal bottom chord A-F-E, a horizontal top chord B-C-D, and a vertical member A-B. Diagonal members connect B-F, F-C, and C-E. A vertical force P is applied at joint F, pointing downwards.</p>
72	 <p>A truss structure with joints A, B, C, D, E, and F. Joint A is a pin support, and joint E is a roller support. The truss consists of a horizontal bottom chord F-E, a horizontal top chord B-C-D, and a vertical member A-F. Diagonal members connect B-F, F-C, and C-D. A horizontal force P is applied at joint B, pointing to the right.</p>
73	 <p>A truss structure with joints A, B, C, D, E, F, and G. Joint A is a pin support, and joint B is a roller support. The truss consists of a horizontal bottom chord A-B, a horizontal top chord F-E-D, and a vertical member G-E. Diagonal members connect A-G, G-B, B-C, C-D, and F-G. A vertical force P is applied at joint E, pointing downwards.</p>